Chapter 7

Imitation Learning

Reading

1. The DAGGER algorithm (https://www.cs.cmu.edu/~sross1/publications/Ross-AIStats11-NoRegret.pdf)
2. https://www.youtube.com/watch?v=TUBBIgtQL_k
3. An Algorithmic Perspective on Imitation Learning (https://arxiv.org/pdf/1811.06711.pdf)

This is the beginning of Module 3 of the course. The previous two modules have been about how to to estimate the state of the world around the robot (Module 1) and how to move the robot (or the world) to a desired state (Module 2). Both of these required that we maintain a model of the dynamics of the robot; this model may be inaccurate and we fudged over this inaccuracy by modeling the remainder as "noise" in Markov Decision Processes.

The next few lectures introduce different aspects of what is called Reinforcement Learning (RL). This is a very large field and you can think of using techniques from RL in many different ways.

1. **Dynamic programming with function approximation.** If we are solving a dynamic programming problem, we can think of writing down the optimal cost-to-go $J^*(x,t)$ as a function of some parameters, e.g., the cost-to-go is

$$J_\varphi(x,t) = \frac{1}{2} x(t)^\top \underbrace{\left(\text{some function of } A, B, Q, R\right)}_{\text{function of } \varphi} x(t)$$

for LQR. We know the stuff inside the brackets to be exactly $P(t)$ but, if we did not, it could be written down as some generic function of parameters φ. We know that any cost-to-go that satisfies the Bellman equation is the optimal cost-to-go, so we can now "fit" the candidate function $J_\varphi(x,t)$ to satisfy the Bellman equation. Similarly, one may also express the optimal feedback control $u(\cdot)$ using some parameters θ as

$$u_\theta(\cdot).$$

We will see how to fit such functions in this chapter.

2. **Learning from data.** It may happen that we do not know very much about the dynamical system, e.g., we do not know a good model for what drives customers as they buy items in an online merchandise platform, or a robot traveling in a crowded area may not have a good model for how large crowds of people walk around it. One may collect data from these systems fit some model of the form $\dot{x} = f(x,u)$ to the data and then go back to the techniques of Module 2. It is typically not clear how much data one should collect. RL gives a suite of techniques to learn the cost-to-go in these situations by collecting and assimilating the data *automatically*. These techniques go under the umbrella of policy gradients, on-policy methods etc. One may also simply "memorize" the data provided by an expert operator, this is called Imitation Learning and we will discuss it next.

Some motivation Imitation Learning is also called "learning from demonstrations". This is in fact one of the earliest successful examples of using a neural network for driving. The ALVINN project at CMU by Dean Pomerleau in 1988 (https://www.youtube.com/watch?v=2KMAAmkz9go) used a two-layer neural network with 5 hidden neurons, about 1000 inputs from the pixels of a camera and 30 outputs. It successfully drove in different parts of the United States and Germany. Imitation learning has also been responsible for numerous other early-successes of RL, e.g., acrobatic maneuvers on an RC helicopter (http://ai.stanford.edu/ acoates/papers/AbbeelCoatesNg_IJRR2010.pdf).

Imitation Learning seeks to record data from experts, e.g., humans, and reproduce these desired behaviors on robots. The key questions we should ask, and which we will answer in this chapter, are as follows.

1. Who should demonstrate (experts, amateurs, or novices) and how should we record data (what states, controls etc.)?
2. How should we learn from this data? e.g., fit a supervised regression model for the policy. How should one ignore bad behaviors in non-expert data?
3. And most importantly, what can we do if the robot encounters a situation which was not in the dataset.

7.1 A crash course in supervised learning

Nature gives us data X and targets Y for this data.

$$X \to Y.$$

Nature does not usually tell us what property of a datum $x \in X$ results in a particular prediction $y \in Y$. We would like to learn to imitate Nature, namely predict y given x.

What does such learning mean? It is simply a notion of being able to identify patterns in the input data without explicitly programming a computer for prediction. We are often happy with a learning process that identifies correlations: if we learn correlations on a few samples $(x^1, y^1), \ldots, (x^n, y^n)$, we may be able to predict the output for a new datum x^{n+1}. We may not need to know *why* the label of x^{n+1} was predicted to be so and so.

Let us say that Nature possesses a probability distribution P over (X, Y). We will formalize the problem of machine learning as Nature drawing n independent and identically distributed samples from this distribution. This is denoted by

$$D_{\text{train}} = \left\{(x^i, y^i) \sim P\right\}_{i=1}^n$$

is called the "training set". We use this data to identify patterns that help make predictions on some future data.

What is the task in machine learning? Suppose D_{train} consists of $n = 50$ RGB images of size 100×100 of two kinds, ones with an orange inside them and ones without. 10^4 is a large number of pixels, each pixel taking any of the possible 255^3 values. Suppose we discover that one particular pixel, say at location $(25, 45)$, takes distinct values in all images inside our training set. We can then construct a predictor based on this pixel. This predictor, it is a binary classifier, perfectly maps the training images to their labels (orange: +1 or no orange: -1). If x^k_{ij} is the $(ij)^{\text{th}}$ pixel for image x^k, then we use the function

❓ How many such binary classifiers are there at most?

$$f(x) = \begin{cases} y^k & \text{if } x^k_{ij} = x_{ij} \text{ for some } k = 1, \ldots, n \\ -1 & \text{otherwise.} \end{cases}$$

This predictor certainly solves the task. It correctly works for all images in the training set. Does it work for images outside the training set?

Our task in machine learning is to learn a predictor that works *outside* the training set. The training set is only a source of information that Nature gives us to find such a predictor.

> Designing a predictor that is accurate on D_{train} is trivial. A hash function that memorizes the data is sufficient. This is NOT our task in machine learning. We want predictors that generalize to new data outside D_{train}.

Generalization If we never see data from outside D_{train} why should we hope to do well on it? The key is the distribution P. Machine learning is formalized as constructing a predictor that works well on new data that is also drawn independently from the distribution P. We will call this set of data the "test set"

$$D_{\text{test}}$$

and it is constructed similarly. This assumption is important. It provides coherence between past and future samples: past samples that were used to train and future samples that we will wish to predict upon. How to find such predictors that work well on new data? The central idea in machine learning is to restrict the set of possible binary functions that we consider.

> We are searching for a predictor that generalizes well but only have the training data to select predictors.

The *right* class of functions f cannot be too large, otherwise we will find our binary classifier above as the solution, and that is not very useful. The class of functions cannot be too small either, otherwise we won't be able to predict difficult images. If the predictor does not even work well on the training set, there is no reason why we should expect it to work on the test set.

> Finding this correct class of functions with the right balance is what machine learning is all about.

❓ Can you now think how is machine learning different from other fields you might know such as statistics or optimization?

7.1.1 Fitting a machine learning model

Let us now solve a classification problem. We will again go around the model selection problem and consider the class of linear classifiers. Assume binary labels $Y \in \{-1, 1\}$. To keep the notation clear, we will use the trick of appending a 1 to the data x and hide the bias term b in the linear classifier. The predictor is now given by

$$\begin{aligned} f(x; w) &= \text{sign}(w^\top x) \\ &= \begin{cases} +1 & \text{if } w^\top x \geq 0 \\ -1 & \text{else.} \end{cases} \end{aligned} \tag{7.1}$$

We have used the sign function denoted as sign to get binary $\{-1, +1\}$ outputs form our real-valued prediction $w^\top x$. This is the famous perceptron model of Frank Rosenblatt.

We want the predictions of the model to match those in the training data and devise an objective to to fit/train the perceptron.

$$\ell_{\text{zero-one}}(w) := \frac{1}{n} \sum_{i=1}^{n} \mathbf{1}_{\{y^i \neq f(x^i; w)\}}. \tag{7.2}$$

The indicator function inside the summation measures the number of mistakes the perceptron makes on the training dataset. The objective here is designed to find weights w that minimizes the average number of mistakes, also known as the training error. Such a loss that measures the mistakes is called the zero-one loss, it incurs a penalty of 1 for a mistake and zero otherwise.

❓ Can you think of some quantity other than the zero-one error that we may wish to optimize?

Surrogate losses The zero-one loss is the clearest indication of whether the perceptron is working well. It is however non-differentiable, so we cannot use powerful ideas from optimization theory to minimize it. This is why surrogate losses are constructed in machine learning. These are proxies for the loss function, typically for the classification problems and look as follows. The exponential loss is

$$\ell_{\text{exp}}(w) = e^{-y\ (w^\top x)}$$

or the logistic loss is

$$\ell_{\text{logistic}}(w) = \log\left(1 + e^{-yw^\top x}\right).$$

Stochastic Gradient Descent (SGD) SGD is a very general algorithm to optimize objectives typically found in machine learning. We can use it so long as we have a dataset and an objective that is differentiable. Consider an optimization problem where we want to solve for

$$w^* = \underset{w}{\text{argmin}}\ \frac{1}{n}\sum_{i=1}^{n} \ell^i(w)$$

where the function ℓ^i denotes the loss on the sample (x^i, y^i) and $w \in \mathbb{R}^p$ denotes the weights of the classifier. Solving this problem using SGD corresponds to iteratively updating the weights using

$$w^{t+1} = w^t - \eta \frac{\mathrm{d}\ell^{\omega_t}(w)}{\mathrm{d}w}\Big|_{w=w^t},$$

i.e., we compute the gradient one sample with index ω_t in the dataset. The index ω_t is chosen uniformly randomly from

$$\omega_t \in \{1, \ldots, n\}.$$

In practice, at each time-step t, we typically select a few (not just one) input data ω_t from the training dataset and average the gradient $\frac{\mathrm{d}\ell^{\omega_t}(w)}{\mathrm{d}w}\Big|_{w=w^t}$ across them; this is known as a "mini-batch". The gradient of the loss $\ell^{\omega_t}(w)$ with respect to w is denoted by

$$\nabla\, \ell^{\omega_t}(w^t) := \frac{\mathrm{d}\ell^{\omega_t}(w)}{\mathrm{d}w}\Big|_{w=w^t} = \begin{bmatrix} \nabla_{w_1}\, \ell^{\omega_t}(w^t) \\ \nabla_{w_2}\, \ell^{\omega_t}(w^t) \\ \vdots \\ \nabla_{w_p}\, \ell^{\omega_t}(w^t) \end{bmatrix} \in \mathbb{R}^p.$$

The gradient $\nabla \ell^{\omega_t}(w^t)$ is therefore a vector in $\mathbb{R}^p$. We have written

$$\nabla_{w_1} \ell^{\omega_t}(w^t) = \frac{\mathrm{d}\ell^{\omega_t}(w)}{\mathrm{d}w_1} \Big|_{w=w^t}$$

for the scalar-valued derivative of the objective $\ell^{\omega_t}(w^t)$ with respect to the first weight $w_1 \in \mathbb{R}$. We can therefore write SGD as

$$w^{t+1} = w^t - \eta \nabla \ell^{\omega_t}(w^t). \tag{7.3}$$

The non-negative scalar $\eta \in \mathbb{R}_+$ is called the step-size or the learning rate. It governs the distance traveled along the negative gradient $-\nabla \ell^{\omega_t}(w^t)$ at each iteration.

7.1.2 Deep Neural Networks

The Perceptron in (7.1) is a linear model: it computes a linear function of the weights $w^\top x$ and uses this function to make the predictions $f(x; w) = \text{sign}(w^\top x)$. Linear models try to split the data (say we have binary labels $Y = \{-1, 1\}$) using a hyper-plane with w denoting the normal to this hyper-plane. This does not work for all situations of course, as the figure below shows, there is no hyper-plane that cleanly separates the two classes (i.e., achieves zero mis-prediction error) but there *is* a nonlinear function that can do the job.

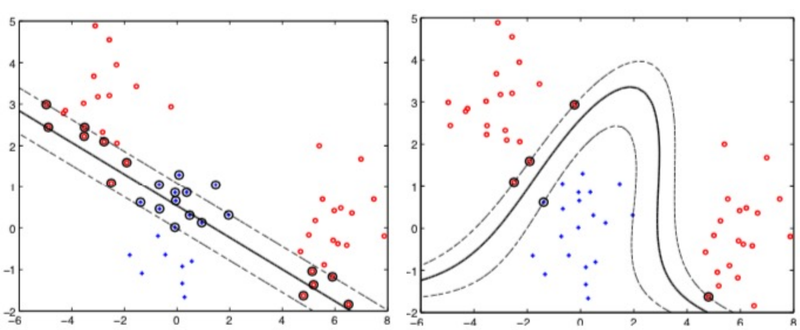


Figure 7.1

A deep neural network is one such nonlinear function. First consider a "two-layer" network

$$f(x; v, S) = \text{sign}\left(v^\top \sigma\left(S^\top x\right)\right)$$

where the matrix $S \in \mathbb{R}^{d\times p}$ and a vector $v \in \mathbb{R}^p$ are the parameters or "weights" of the classifier. The "nonlinearity" σ is usually set to be what is called a Rectified Linear Unit (ReLU)

$$\begin{aligned} \sigma(x) := \text{ReLU}(x) &= |x|_+ \\ &= \max(0, x). \end{aligned} \tag{7.4}$$

Just like the case of a Perceptron, we can use an objective $\frac{1}{n}\sum_{i=1}^n \ell^i(v, S)$

that depends on both v, S to fit this classifier on training data. A deep neural network takes the idea of a two-layer network to the next step and has multiple "layers", each with a different weight matrix $S_1, \ldots, S_L$. The classifier is therefore given by

$$f(x; v, S_1, \ldots, S_L) = \text{sign}\left(v^\top \sigma\left(S_L^\top \ldots \sigma\left(S_2^\top\, \sigma(S_1^\top x)\right) \ldots\right)\right). \qquad (7.5)$$

We call each operation of the form $\sigma\left(S_k^\top \ldots\right)$, as a *layer*. Consider the second layer: it takes the features generated by the first layer, namely $\sigma(S_1^\top x)$, multiplies these features using its feature matrix $S_2^\top$ and applies a nonlinear function $\sigma(\cdot)$ to this result element-wise before passing it on to the third layer.

A deep network creates new features by composing older features.

This composition is very powerful. Not only do we not have to pick a particular feature vector, we can create very complex features by sequentially combining simpler ones. For example Figure 7.2 shows the features (more precisely, the kernel) learnt by a deep neural network. The first layer of features are called Gabor-like, and incidentally they are similar to the features learned by the human brain in the first part of the visual cortex (the one closest to the eyes). These features are *combined* linearly along with a nonlinear operation to give richer features (spirals, right angles) in the middle panel. The third layer combines the lower features to get even more complex features, these look like patterns (notice a soccer ball in the bottom left), a box on the bottom right etc.

Deep networks are universal function approximators The multi-layer neural network is a powerful class of classifiers: depending upon how many layers we have and what is the dimensionality of the the weight matrices S_k at each layer, we can fit *any* training data. In fact, this statement, which is called the *universal approximation property* holds even for a two-layer neural network $v^\top \sigma(S^\top x)$ if the number of columns in S is big enough. This property is the central reason why deep networks are so widely applicable, we can model complex machine learning problems if we choose a big enough deep network.

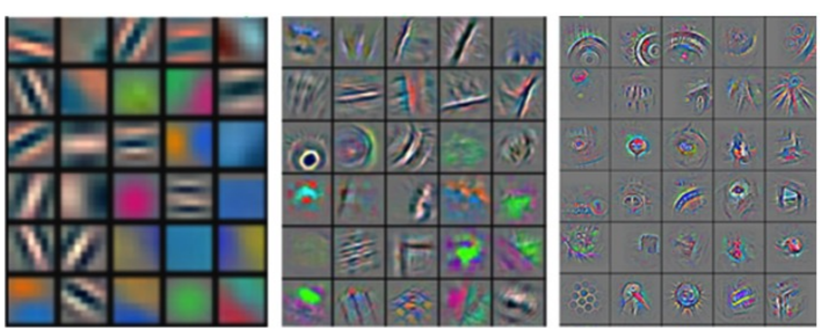


Figure 7.2

Logits for multi-class classification. The output

$$\hat{y} = v^\top \sigma\left(S_L^\top \ldots \sigma\left(S_2^\top\ \sigma(S_1^\top x)\right) \ldots\right)$$

is called the logits corresponding to the different classes. This name comes from logistic regression where logits are the log-probabilities of an input datum belonging to one of the two classes. A deep network provides an easy way to solve a multi-class classification problem, we simply set

$$v \in \mathbb{R}^{p \times C}$$

where C is the total number of classes in the data. Just like logistic regression predicts the logits of the two classes, we would like to *interpret* the vector $\hat{y}$ as the log-probabilities of an input belonging to one of the classes.

❓ What would the shape of w be if you were performing regression using a deep network?

Weights It is customary to not differentiate between the parameters of different layers of a deep network and simply say *weights* when we want to refer to all parameters. The set

$$w := \{v, S_1, S_2, \ldots, S_L\}$$

is the set of *weights*. This set is typically stored in PyTorch as a set of matrices, one for each layer. Using this new notation, we will write down a deep neural network classifier as simply

$$f(x, w) \tag{7.6}$$

and fitting the deep network to a dataset involves the optimization problem

$$w^* = \underset{w}{\text{argmin}}\ \frac{1}{n} \sum_{i=1}^{n} \ell(y^i, \hat{y}^i). \tag{7.7}$$

We will also sometimes denote the loss of the i^{th} sample as

$$\ell^i(w) := \ell(y^i, \hat{y}^i).$$

Backpropagation The Backpropagation algorithm is a method to compute the gradient of the objective while fitting a deep network using SGD, i.e., it computes $\nabla_w\, \ell^i(w)$. For the purposes of this course, the details of how this is done are not essential, so we will skip them.

PyTorch We will use a library called PyTorch (https://pytorch.org) to code up deep neural networks for the reinforcement learning part of this course. You can find some excellent tutorials for it at https://pytorch.org/tutorials/beginner/basics/intro.html. For the purposes of this course, you do not need to know the intricacies of PyTorch, we will give you enough code to work with deep networks so that you can focus on implementing the reinforcement learning-specific parts.

7.2 Behavior Cloning

With that background, we are ready to tackle what is potentially the simplest problem in RL. We will almost exclusively deal with discrete-time systems for RL. Let us imagine that we are given access to n trajectories each of length $T+1$ time-steps from an expert demonstrator for our system. We write this as a training dataset

$$D = \left\{ (x_t^i, u_t^i)_{t=0,1,\ldots,T} \right\}_{i=1,\ldots,n}$$

At each step, we record the state $x_t^i \in \mathbb{R}^d$ and the control that the expert took at that state u_t^i. We would like to learn a deterministic feedback control for the robot that is parametrized by parameters θ

$$u_\theta(x) : X \mapsto U \subset \mathbb{R}^m.$$

using the training data. The idea is that if $u_\theta(x^i(t)) \approx u^i(t)$ for all i and all times t, then we can simply run our learned controller $u_\theta(x)$ on the robot instead of having the expert. A simple example is a baby deer learning to imitate how its mother in how to run.

Parameterizing the controller Our function u_θ may represent many different families of controllers. For example, $u_\theta(x) = \theta x$ where $\theta \in \mathbb{R}^{d \times p}$ is a linear controller; this is much like the control for LQR except that we can fit θ to the expert's data instead of solving the LQR problem to find the Kalman gain. We could also think of some other complicated function, e.g., a two-layer neural network,

$$u_\theta(x) = v \ \sigma \left(S^\top x \right)$$

where $S \in \mathbb{R}^{d \times p}$ and $v \in \mathbb{R}^{m \times p}$ and $\sigma : \mathbb{R}^m \mapsto \mathbb{R}^m$ is some nonlinearity, say ReLU. As we did above, we will use

$$\theta := (v, S)$$

to denote all the weights of this two-layer neural network. Multi-layer neural networks are also another possible avenue. In general, we want to the parameterization of the controller to be rich enough to fit some complex controller that the expert may have used on the system.

How to fit the controller? Given our chosen model for $u_\theta(x)$, say a two-layer neural network with weights θ, fitting the controller involves finding the best value for the parameters θ such that $u_\theta(x_t^i) \approx u_t^i$ for data in our dataset. There are many ways to do this, e.g., we can solve the following optimization problem

$$\widehat{\theta} = \underset{\theta}{\text{argmin}} \ \ell(\theta) := \frac{1}{n} \sum_{i=1}^{n} \underbrace{\frac{1}{T+1} \sum_{t=0}^{T} \| u_t^i - u_\theta(x_t^i) \|_2^2}_{\ell^i(\theta)} \tag{7.8}$$

The difficulty of solving the above problem depends upon how difficult the model $u_\theta(x)$ is, for instance, if the model is linear $\theta\ x$, we can solve (7.8) using ordinary least squares. If the model is a neural network, one would have to use SGD to solve the optimization problem above. After fitting this model, we have a new controller

$$u_{\widehat{\theta}}(x) \in \mathbb{R}^m$$

that we can use *anywhere* in the domain $X \subset \mathbb{R}^d$, even at places where we had no expert data. This is known as Behavior Cloning, i.e., cloning the controls of the expert into a parametric model.

Generalization performance of behavior cloning Note that the data provided by the expert is not iid, of course the state x^i_{t+1} in the expert's trajectory depends upon the previous state x^i_t. Standard supervised learning makes the assumption that Nature gives training data that is independent and identically distributed from the distribution P. While it is still reasonable to fit the regression loss in (7.8) for such correlated data, one should remember that if the expert trajectories do not go to all parts of the state-space, the learned controller fitted on the training data may not work outside these parts. Of course, if we behavior clone the controls taken by a generic driver, they are unlikely to be competitive for racing, and vice-versa. It is very important to realize that this does *not* mean that BC does not generalize. Generalization in machine learning is a concept that suggests that the model should work well on data *from the same distribution*. What does the the "distribution" of the expert mean, in this case, it simply refers to the distribution of the states that the expert's trajectories typically visit, e.g, a race driver typically drives at the limits of tire friction and throttle, this is different from a usual city-driver who would rather maximize the longevity of their tires and engine-life.

ℹ Discuss generalization performance in behavior cloning.

7.2.1 Behavior cloning with a stochastic controller

In this case, we have always chosen feedback feedback controllers that are deterministic, i.e., there is a single value of control u that is taken at the state x. Going forward, we will also talk about stochastic controllers, i.e., controllers which sample a control from a distribution. There can be a few reasons of using such a controller. First, we will see in later lectures how this may help in training a reinforcement learning algorithm; this is because in situations where you do not know the system dynamics precisely, it helps to "hedge" the feedback to take a few different control actions instead of simply the one that the value function deems as the maximizing one. This is not very different from having a few different stocks in your portfolio. Second, we benefit from this hedging even at test-time when we run a stochastic feedback control, e.g., in situations where the limited training data may not want to always pick the best control (because the best control was computed using an imprecise model of the system dynamics and could be wrong), but rather hedge our bets by choosing between a few different controls.

A stochastic feedback control is denoted by

$$u \sim u_\theta(\cdot \mid x) = \mathrm{P}(\cdot \mid x)$$

notice that $u_\theta(\cdot \mid x)$ is a probability distribution on the control space U that depends on the state x, and in this case the parameters θ. The control taken at a state x is a sample drawn from this probability distribution. The deterministic controller is a special case of this setup where

$$u_\theta(u|\, x) = \delta_{u_\theta(x)}(u) \equiv u_\theta(x)$$

is a Dirac-delta distribution at $u_\theta(x)$. If the control space U is discrete, then $u_\theta(\cdot \mid x)$ could be a categorical distribution. If the control space U is continuous, then you may wish to think of the controls being sampled from a Gaussian distribution with some mean $\mu_\theta(x)$ and variance $\sigma^2_\theta(x)$

$$\mathbb{R}^m \ni u \sim u_\theta(\cdot \mid x) = N(\mu_\theta(x), \Sigma_\theta(x)).$$

Maximum likelihood estimation Let's pick a particular stochastic controller, say a Gaussian. How should we fit the parameters θ for this? We would like to find parameters θ that make the expert's data in our dataset very likely. The log-likelihood of each datum is

$$\log u_\theta(u^i_t \mid x^i_t)$$

and maximizing the log-likelihood of the entire dataset amounts to solving

$$\widehat{\theta} = \underset{\theta}{\mathrm{argmin}} \, \frac{1}{n} \sum_{i=1}^{n} \underbrace{\frac{1}{T+1} \sum_{t=0}^{T} - \log u_\theta(u^i_t \mid x^i_t)}_{\ell^i(\theta)} . \tag{7.9}$$

Fitting BC with a Gaussian controller Notice that if we use a Gaussian distribution

$$u_\theta(\cdot \mid x) = N\left(\mu_\theta(x), I\right)$$

as our stochastic controller, the objective in (7.9) is the same as that in (7.8).

$$u_\theta(\cdot \mid x) = N\left(\mu_\theta(x), \sigma^2_\theta(x) I\right)$$

we have that

$$-\log u_\theta(u \mid x) = \frac{\|\mu_\theta(x) - u\|_2^2}{\sigma^2_\theta(x)} + 2cp\, \log \sigma_\theta(x).$$

where c is a constant.

7.2.2 KL-divergence form of Behavior Cloning

Background on KL divergence The Kullback-Leibler (KL) divergence is a quantity to measure the distance between two probability distributions. There are many similar distances, for example, given two probability distributions

$p(x)$ and $q(x)$ supported on a discrete set X, the total variation distance between them is

$$\text{TV}(p, q) = \frac{1}{2} \sum_{x \in X} |p(x) - q(x)| \, .$$

Hellinger distance (https://en.wikipedia.org/wiki/Hellinger_distance), f-divergences (https://en.wikipedia.org/wiki/F-divergence) and the Wasserstein metric (https://en.wikipedia.org/wiki/Wasserstein_metric) are a few other examples of ways to measure how different two probability distributions are from each other.

The Kullback-Leibler divergence (KL) between two distributions is given by

$$\text{KL}(p \mid\mid q) = \sum_{x \in X} p(x) \log \frac{p(x)}{q(x)}. \tag{7.10}$$

This is a distance and not a metric, i.e., it is always non-zero and zero if and only if the two distributions are equal, but the KL-divergence is not symmetric (like a metric has to be). Also, the above formula is well-defined only if for all x where $q(x) = 0$, we also have $p(x) = 0$. Notice that it is not symmetric

$$\text{KL}(q \mid\mid p) = \sum_{x \in X} q(x) \log \frac{q(x)}{p(x)} \quad \neq \text{KL}(p \mid\mid q).$$

The funny notation $\text{KL}(p \mid\mid q)$ was invented by Shun-ichi Amari (https://en.wikipedia.org/wiki/Shun%27ichi_Amari) to emphasize the fact that the KL-divergence is asymmetric. The KL-divergence is always positive: you can show this using an application of Jensen's inequality. For distributions with continuous support, we integrate over the entire space X and define KL divergence as

$$\text{KL}(p \mid\mid q) = \int_{\mathcal{X}} p(x) \log \frac{p(x)}{q(x)} \, \text{d}x.$$

Behavior Cloning Let us now imagine the expert is also a parametric stochastic feedback controller $u_{\theta^*}(\cdot \mid x)$. Our data is therefore drawn by running this controller for n trajectories, T time-steps on the system. This dataset now consists of samples from

$$p_{u_{\theta^*}}(x, u)$$

which is the joint distribution on the state-space X and the control-space U. We have denoted the parameters of the feedback controller which creates this distribution as the subscript u_{θ^*}. Our behavior cloning controller creates a similar distribution $p_{u_\theta}(x, u)$ and the general version of the objective in (7.9) is therefore

$$\widehat{\theta} = \underset{\theta}{\text{argmin}} \, \text{KL} \left(p_{u_{\theta^*}} \mid\mid p_{u_\theta} \right); \tag{7.11}$$

The objective in (7.9) corresponds to this for Gaussian stochastic controllers, but we can just as easily imagine some other distribution for the stochastic controller of the expert and the robot.

Written this way, BC can be understood as finding a controller $\widehat{\theta}$ whose distribution on the states and controls is close to the distribution of states and controls of the expert.

7.2.3 Some remarks on Behavior Cloning

Worst-case performance Performance of Behavior Cloning can be quite bad in the worst case. The authors in "Efficient reductions for imitation learning" (https://www.cs.cmu.edu/~sross1/publications/Ross-AIStats11-NoRegret.pdf) show that if the learned controller $u_{\widehat{\theta}}$ differs from the control taken by the expert controller u_{θ^*} with a probability ϵ at each time-step, over a horizon of length T time-steps, it can be $\mathcal{O}(T^2\epsilon)$ off from the cost-to-go of the expert *as averaged over states that the learned controller visits*. This is because once the robot makes a mistake and goes away from the expert's part in the state-space, future states of the robot and the expert can be very different.

ℹ Draw a picture of the amplifying errors of running behavior cloning in real-time.

Model-free nature of BC Observe that our learned controller $u_{\widehat{\theta}}(\cdot \mid x)$ is a feedback controller and works for entire state-space X. We did not need to know the dynamics of the system to build this controller. The data from the expert is conceptually the same as the model $\dot{x} = f(x, u)$ of the dynamics, and you can learn controllers from both. Do you however notice a catch?

7.3 DAgger: Dataset Aggregation

The expert's dataset in Behavior Cloning determines the quality of the controller learned. If we collected very few trajectories from the expert, they may not cover all parts of the state-space and the behavior cloned controller has no data to fit the model in those parts.

Let us design a simple algorithm, of the same spirit as iterative-LQR, to mitigate this. We start with a candidate controller, say $u_{\theta^{(0)}}(x)$; one may also start with a stochastic controller $u_{\theta^{(0)}}(\cdot \mid x)$ instead.

DAgger: Let the dataset $D^{(0)}$ be the data collected from the expert.

Initialize $u_{\theta^{(0)}} = u_{\widehat{\theta}}$ to be the BC controller learned using data $D^{(0)}$. At iteration k

1. The robot queries the expert for a fraction p of the time-steps and uses its learned controller $u_{\theta^{(k-1)}}$ for the other time-steps. If the expert corresponds to some controller u_{θ^*}, then the robot controller at a state x is

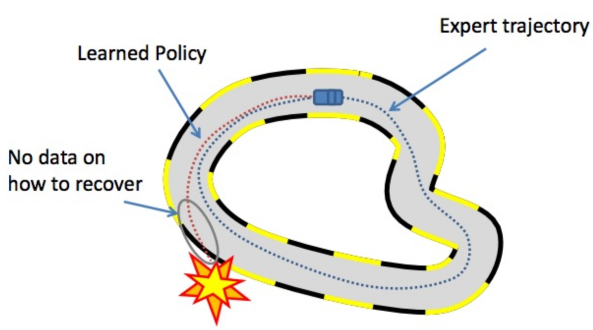
$$u \sim p\, \delta_{u_{\theta^*}(x)} \, + (1-p)\, \delta_{u_{\theta^{(k-1)}}(x)}.$$

2. Use $u(x)$ to collect a dataset $D = \left\{ (x_t^i, u_t^i)_{t=0,\dots,T} \right\}_{i=1,\dots,n}$ with n trajectories.

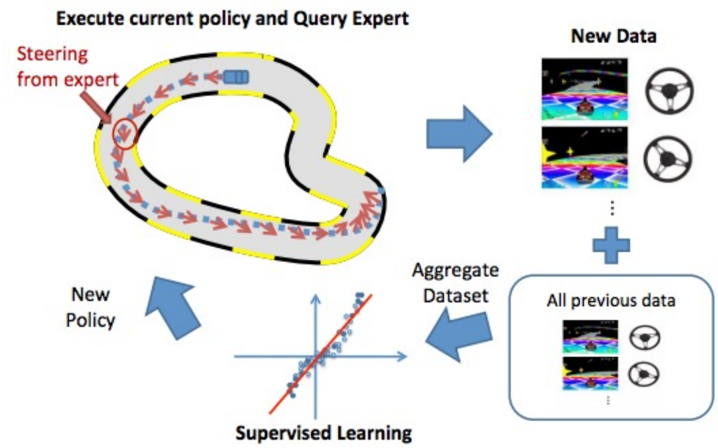
3. Set the new dataset to be $D^{(k)} = D^{(k-1)} \cup D$

4. Fit a controller $u_{\theta^{(k)}}$ using behavior cloning to the new dataset $D^{(k)}$.

The above algorithm iteratively updates the BC controller $u_{\widehat{\theta}}$ by drawing new data from the expert. The robot first bootstraps off the expert's data, this simply means that it uses the expert's data to fit its controller $u_{\theta^{(0)}}(x)$. As we discussed above, this controller may veer off the expert's trajectory if the robot starts at states that are different from the dataset, or even if it takes a slightly different control than the expert midway through a trajectory.



To fix this, the robot collects more data at each iteration. It uses a combination of the expert and its controller to collect such data. This, allows *collecting a dataset of expert's controls in states that the robot visits* and iteratively expands the dataset $D^{(k)}$.



In the beginning we may wish to be close to the expert's data and use a large value of p, as the fitted controller $u_{\theta_{k+1}}$ becomes good, we can reduce the value of p and rely less on the expert.

DAgger is an iterative algorithm which expands the controller to handle larger and larger parts of the state-space. Therefore, the cost-to-go of the controller learned via DAgger is $\mathcal{O}(T)$ off from the cost-to-go of the expert *as averaged over states that the learned controller visits.*

What criterion can we use to stop these iterations? We can stop when the incremental dataset collected D_k is not that different from the cumulative dataset D, we know that the new controllers are not that different. We can also stop when the parameters of our learned controller are $\theta^{(k+1)} \approx \theta^{(k)}$.

DAgger with expert annotations at each step DAgger is a conceptual framework where the expert is queried repeatedly for new control actions. This is obviously problematic because we need to expert on hand at each iteration. We can also cook up a slightly version of DAgger where we start with the BC controller $u_{\theta^{(k)}} = u_{\widehat{\theta}}$ and at each step, we run the controller on the real system and ask the expert to relabel the data after that run. The dataset $D^{(k)}$ collected by the algorithm expands at each iteration and although the states x_t^i are those visited by our controller, their annotations are those given by the expert. This is a much more natural way of implementing DAgger.