Model Predictive Control

Lecture 9

Principles of Modeling for Cyber-Physical Systems

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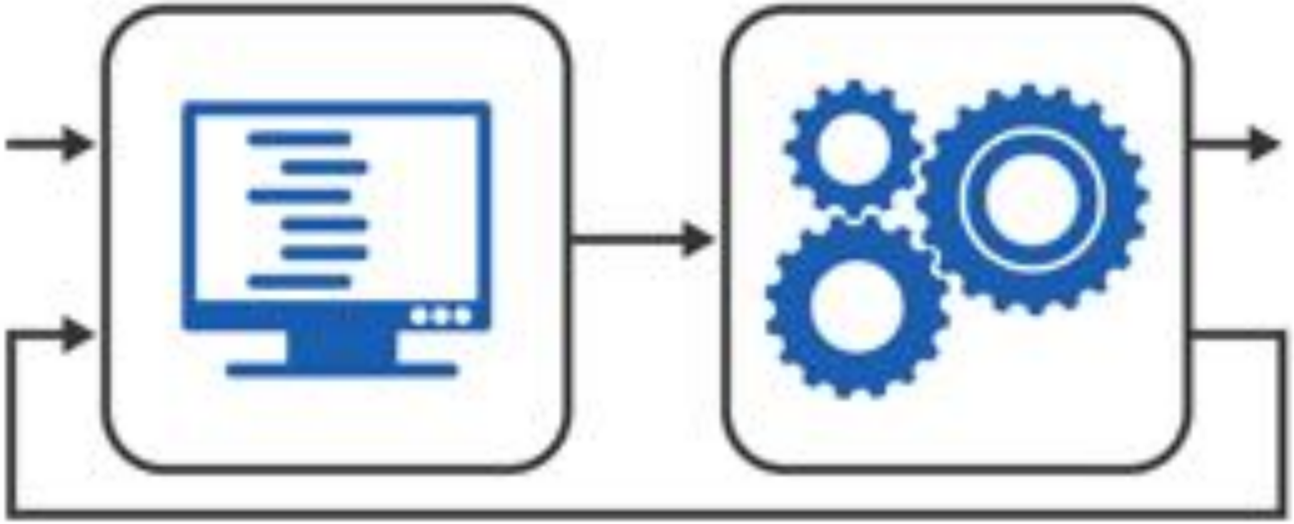
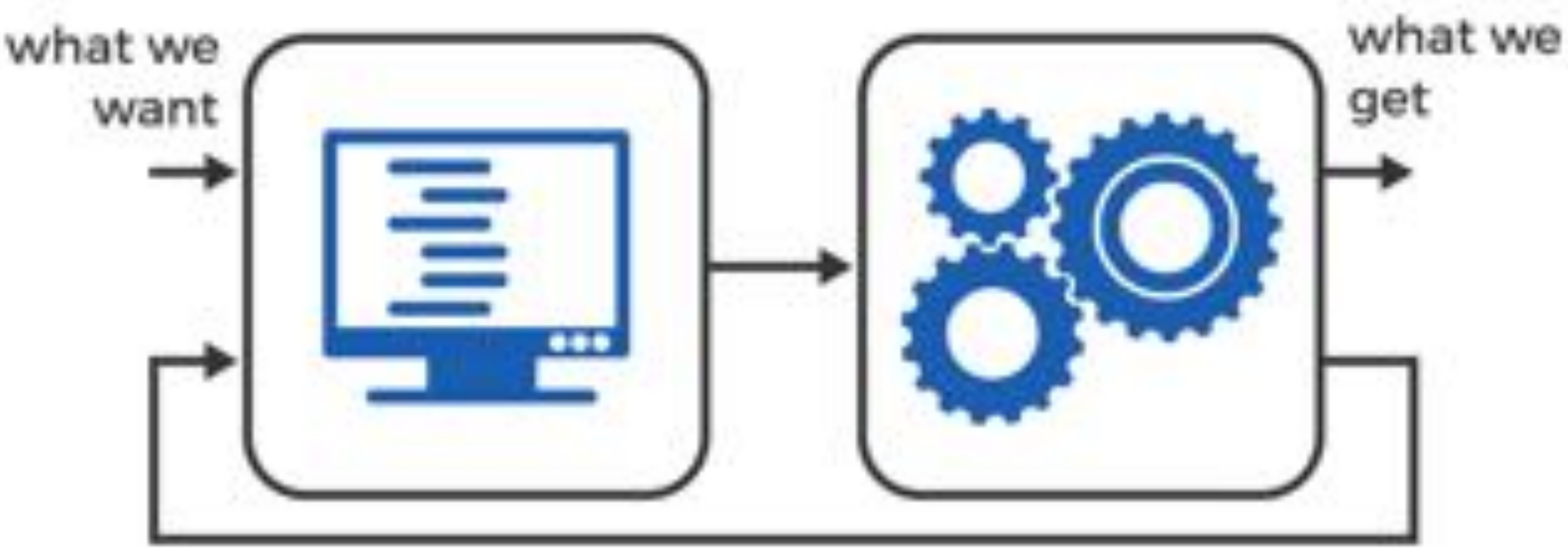
Control


make the system
behave like we want

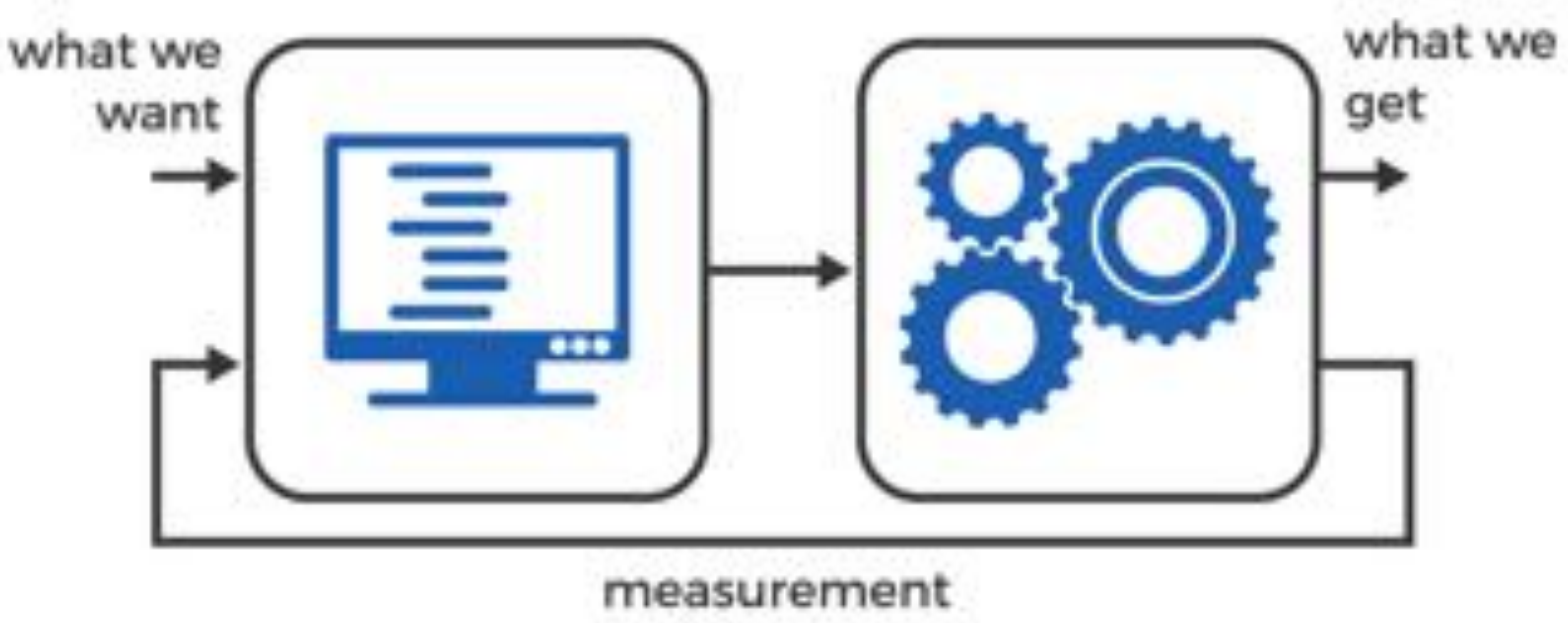
Control



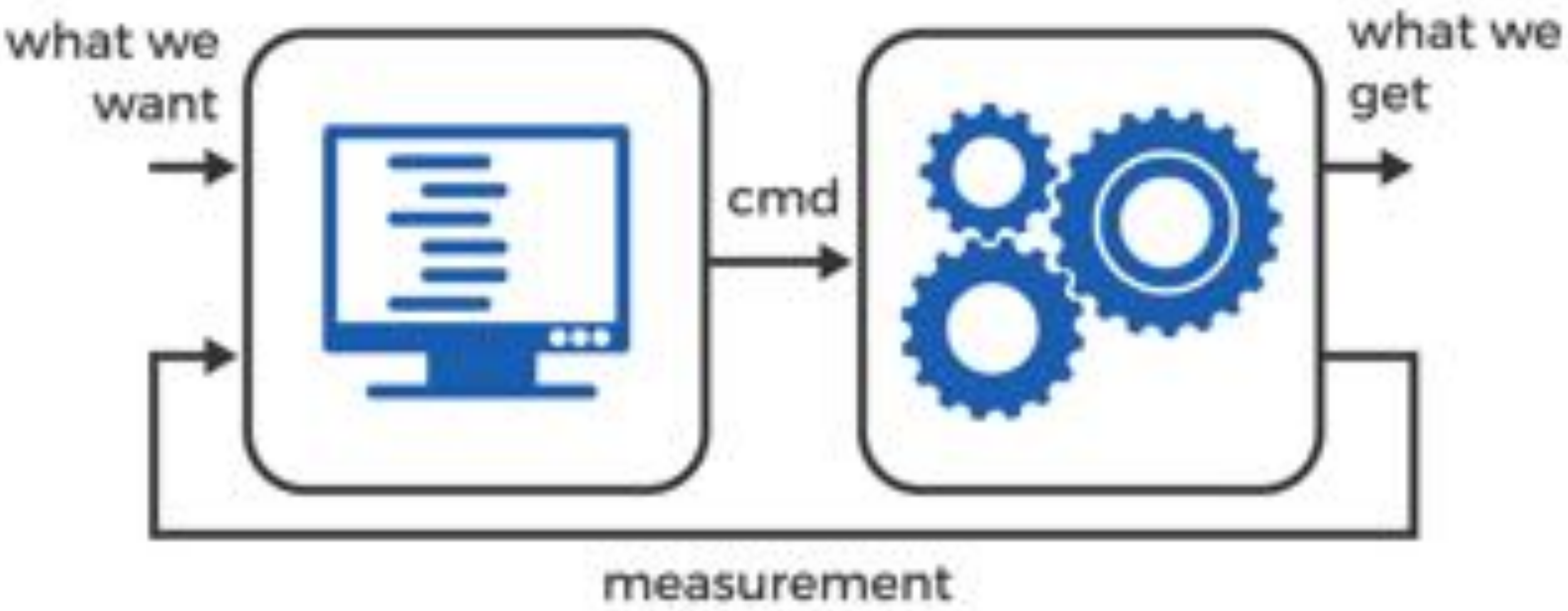
Control



Control



Control



Control

Control

Control


desired temperature
23
real temperature
temperature measurement

Control


desired temperature
heater on/off
23
real temperature
temperature measurement

Control

measure

Control

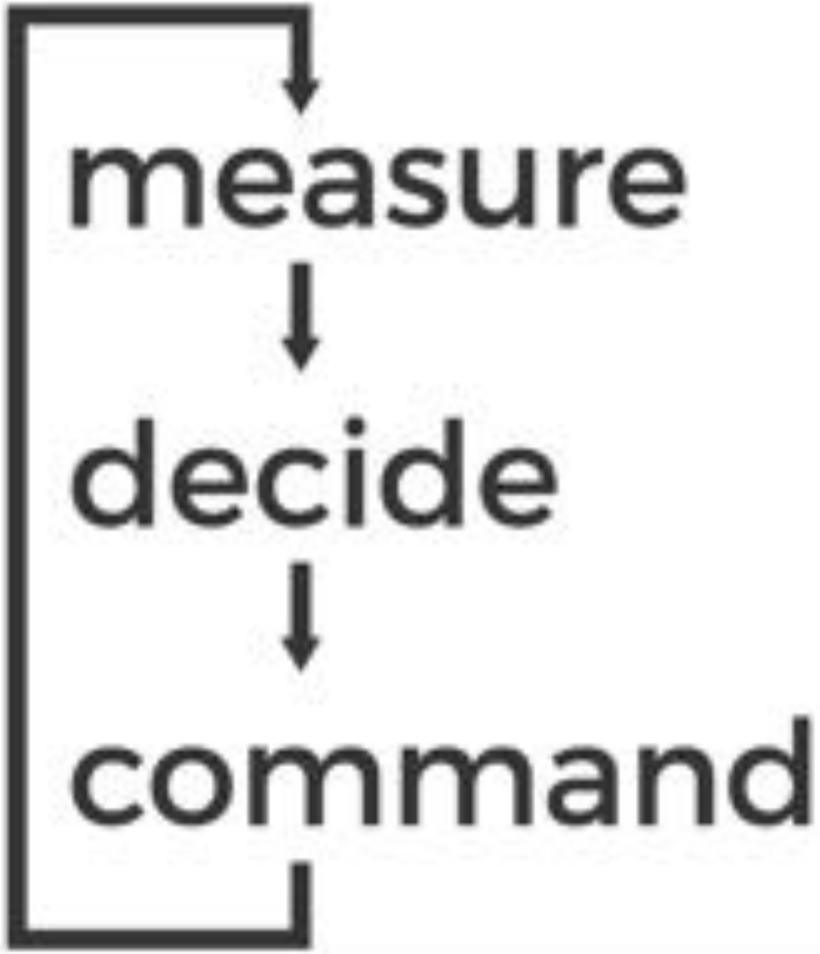
measure

↓

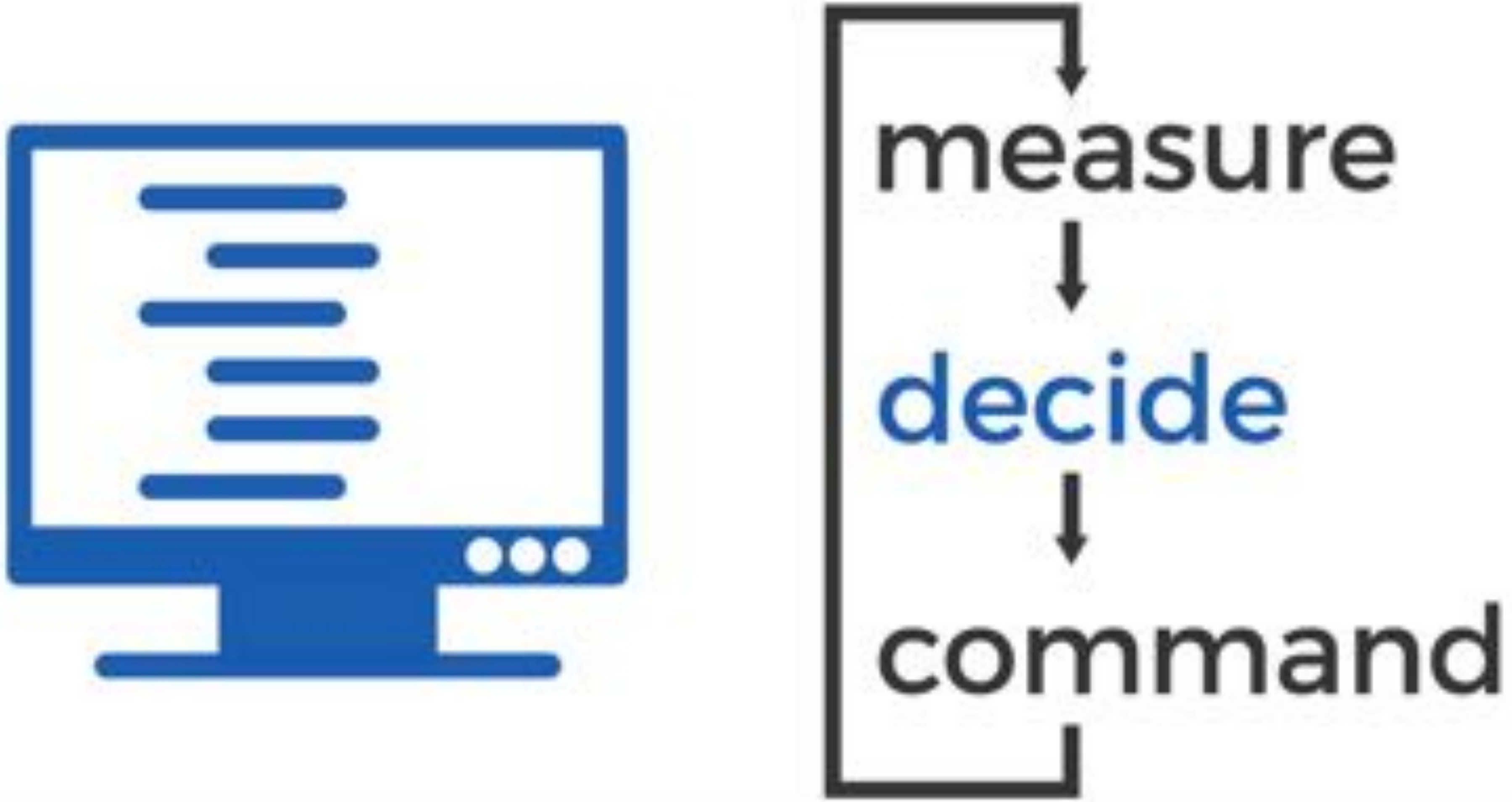
decide

Control

Control



Control



Control


```
if temperature too low then
     turn on heater
if temperature too high then
     turn off heater
```

Model predictive control

Model predictive control

- Decision based on prediction of system's behavior

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Geek+ 70421
SUNING 苏宁物流

Model predictive control

Model predictive control

without prediction

Model predictive control

without prediction

Model predictive control

without prediction

Model predictive control

without prediction

Model predictive control

without prediction

Model predictive control

without prediction

Model predictive control

without prediction

Model predictive control

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Model predictive control

with prediction

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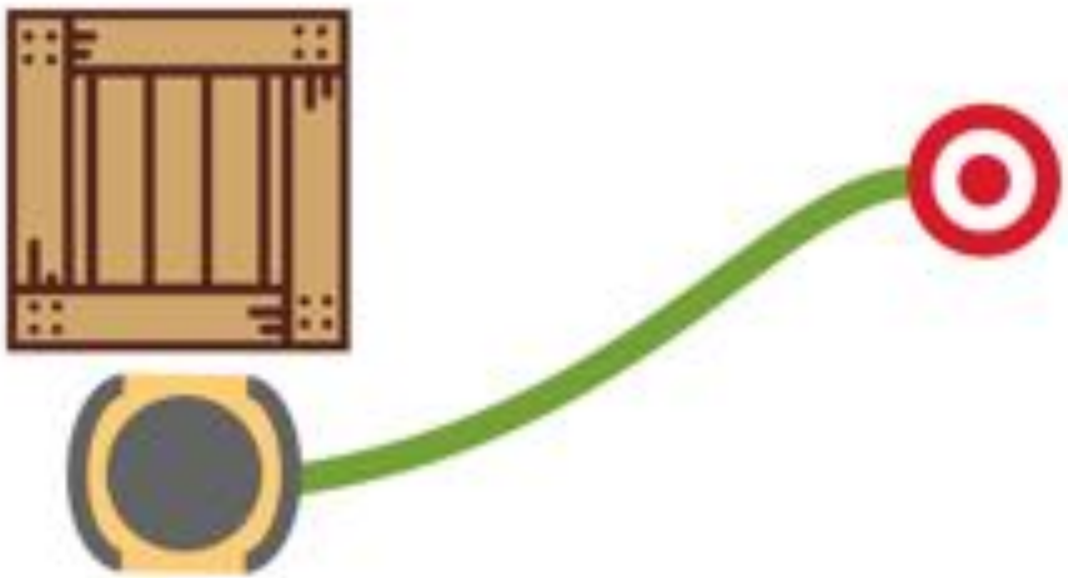
with prediction

Model predictive control

with prediction



Model predictive control

with prediction

Model predictive control

with prediction

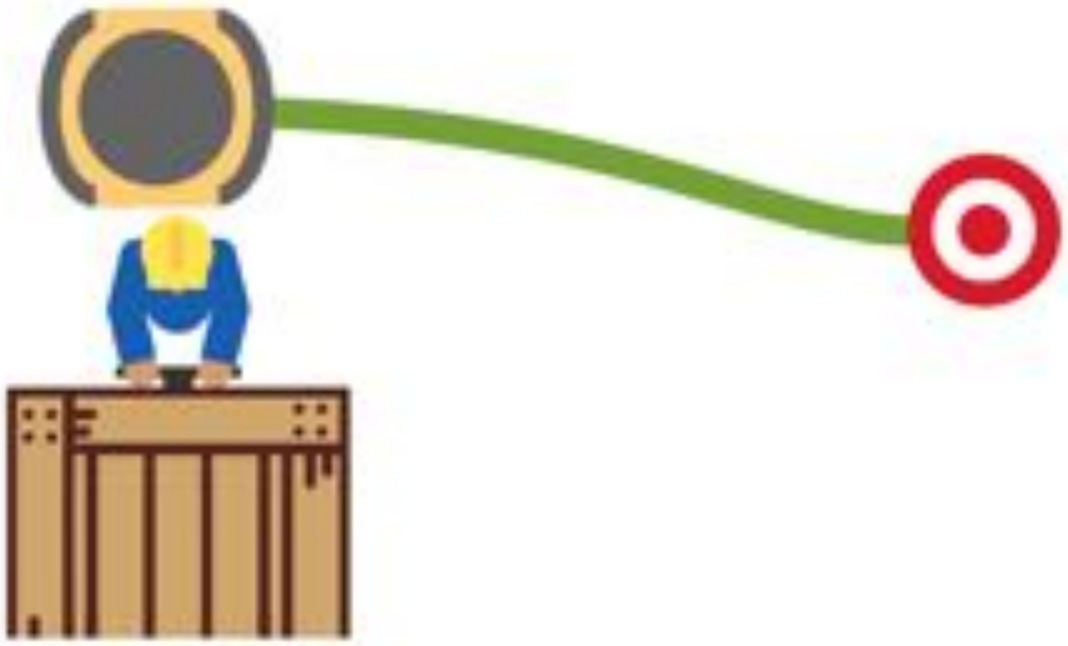

Model predictive control
with prediction

Model predictive control

with prediction

Model predictive control

with prediction



Model predictive control

with prediction

Model predictive control

- Decision based on prediction of system's behavior
- Decision made using optimization

Model predictive control

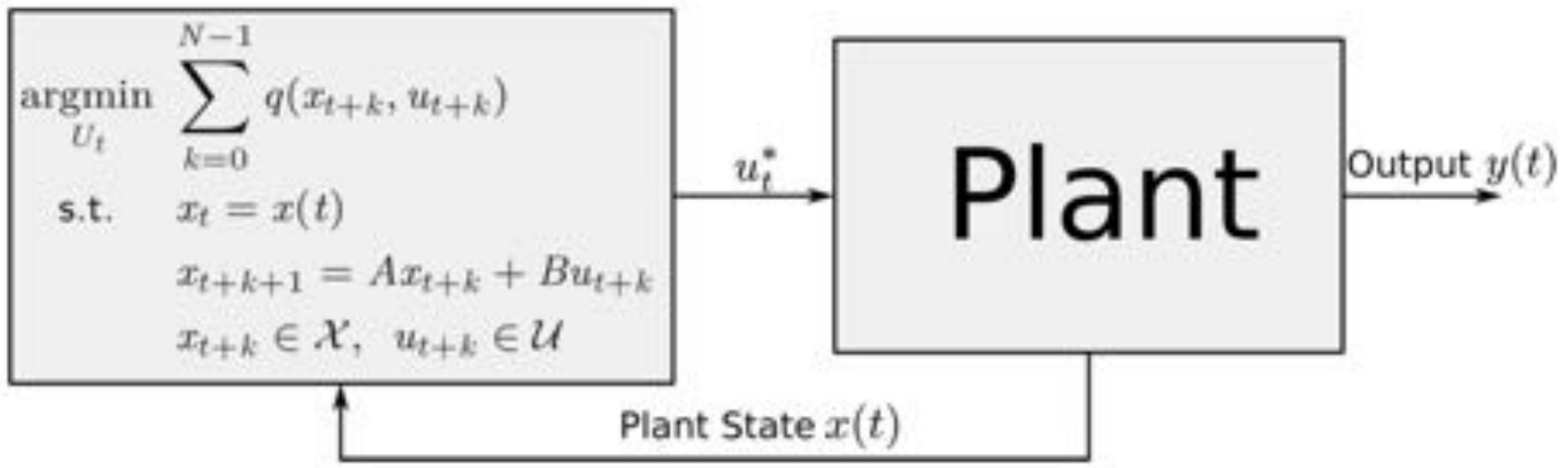
make optimal decision

Model predictive control

optimal decision

Model predictive control

MPC: Mathematical formulation



MPC: Mathematical formulation

$$
\begin{aligned}
U_t^*(x(t)) := \underset{U_t}{\operatorname{argmin}} & \sum_{k=0}^{N-1} q(x_{t+k}, u_{t+k}) && \\
\text{subj. to } & x_t = x(t) && \text{measurement} \\
& x_{t+k+1} = Ax_{t+k} + Bu_{t+k} && \text{system model} \\
& x_{t+k} \in \mathcal{X} && \text{state constraints} \\
& u_{t+k} \in \mathcal{U} && \text{input constraints} \\
& U_t = \{u_t, u_{t+1}, \ldots, u_{t+N-1}\} && \text{optimization variables}
\end{aligned}
$$

Receding horizon philosophy

- MPC is like **playing chess** !

MPC: Mathematical formulation

At each sample time:

- Measure / estimate current state $x(t)$
- Find the optimal input sequence for the entire planning window N:
  $U_t^* = \{u_t^*, u_{t+1}^*, \ldots, u_{t+N-1}^*\}$
- Implement only the *first* control action u_t^*

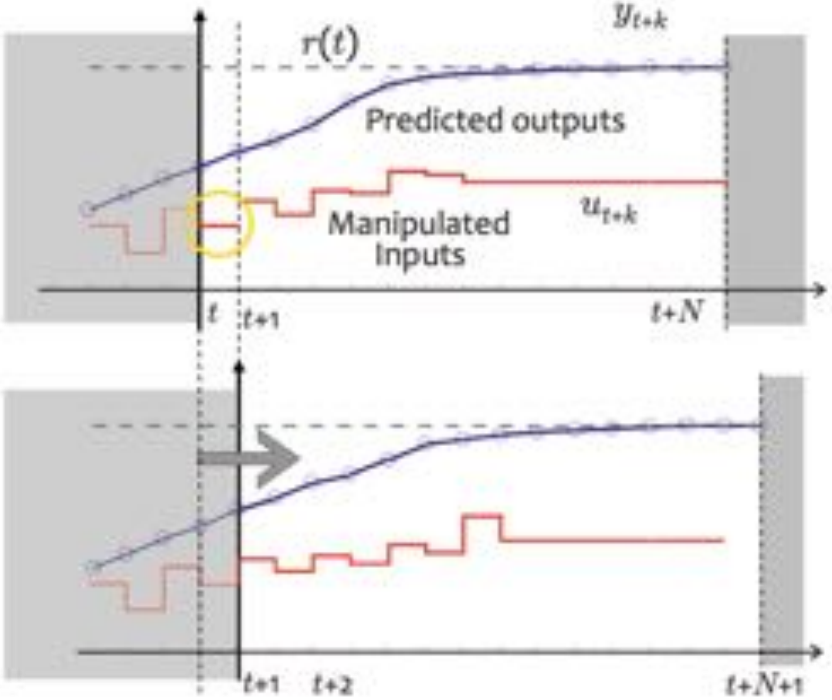
Receding horizon philosophy

- At time t: solve an **optimal control** problem over a finite future horizon of N steps:

$$\min_{u_t,\ldots,u_{t+N-1}} \left\{ \sum_{k=0}^{N-1} \|y_{t+k} - r(t)\|^2 + \rho\|u_{t+k} - u_r(t)\|^2 \right\}$$

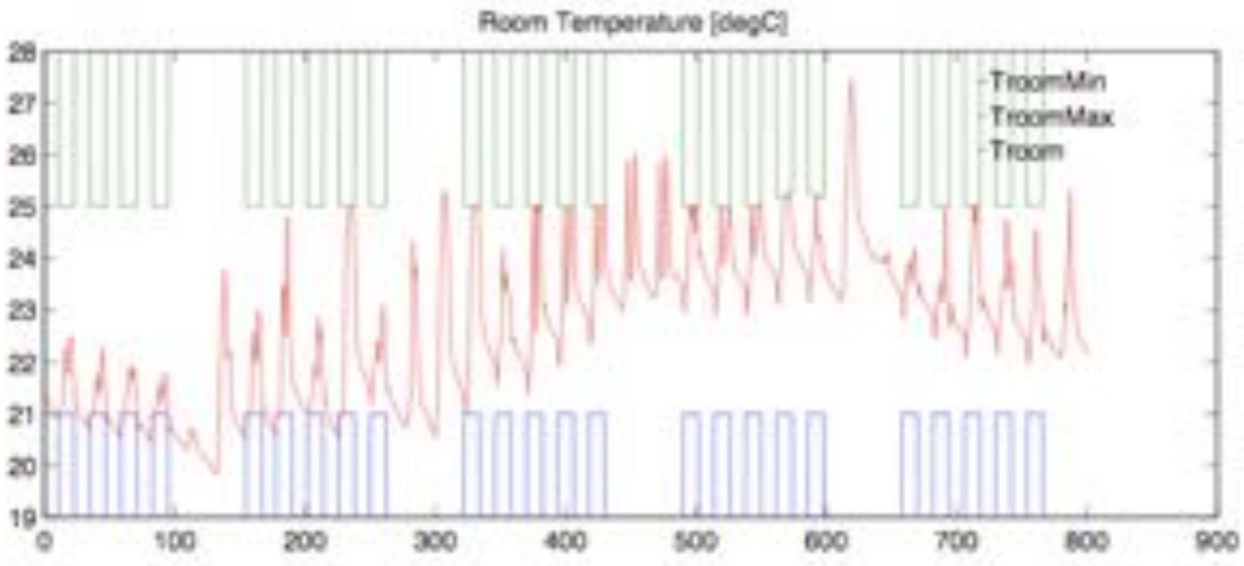
$$\begin{aligned}
\text{s.t.} \quad & x_{t+k+1} = f(x_{t+k}, u_{t+k}) \\
& y_{t+k} = g(x_{t+k}, u_{t+k}) \\
& u_{min} \leq u_{t+k} \leq u_{max} \\
& y_{min} \leq y_{t+k} \leq y_{max} \\
& x_t = x(t), \ k = 0, \ldots, N-1
\end{aligned}$$

- Only apply the first optimal move $u^*(t)$



Energy Efficient Building Control

Control Task: Use minimum amount of energy (or money) to keep room temperature, illuminance level and CO_2 concentration in *prescribed comfort ranges*



[*OptiControl Project, ETH. 2010;* http://www.opticontrol.ethz.ch/]

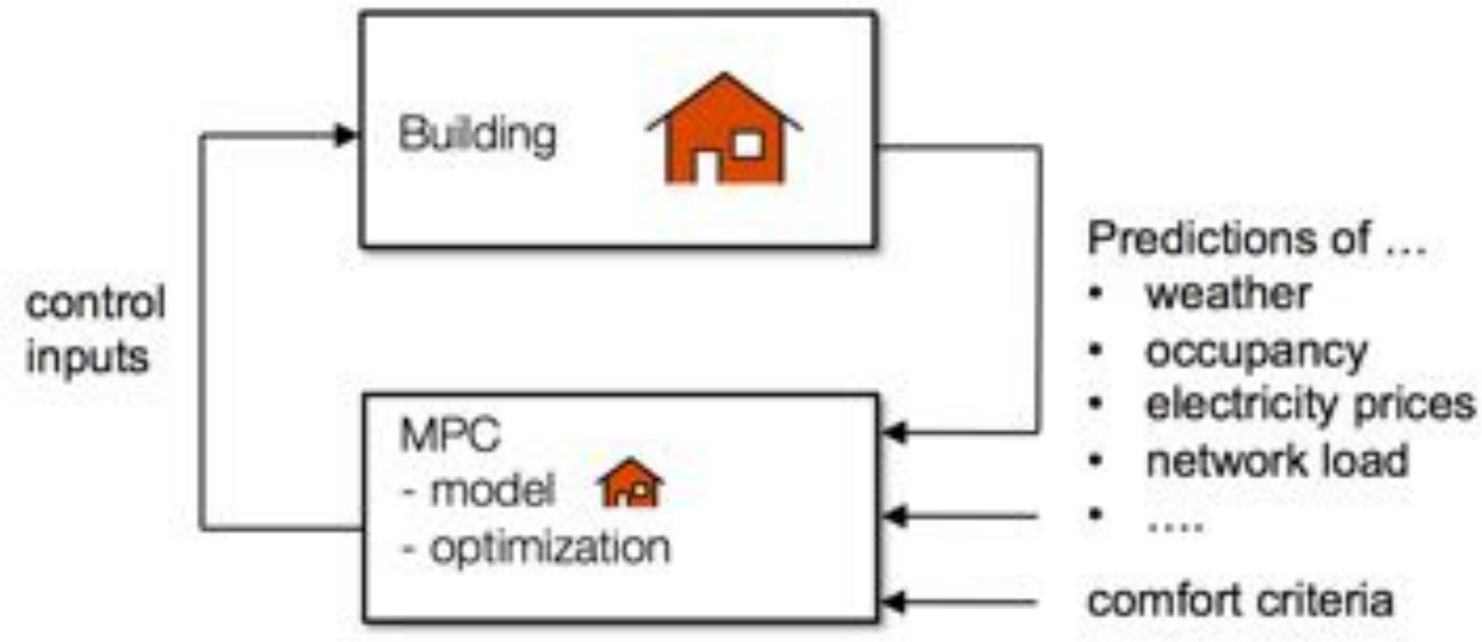
Energy Efficient Building Control

MPC opens the possibility to

- exploit building's *thermal storage capacity*
- use *predictions* of future disturbances, e.g. weather, for better planning
- use forecasts of electricity prices to shift electricity demand for grid-friendly behavior
- offer grid-balancing services to the power network
- ...

while respecting requirements for building usage (temperature, light, ...)

Energy Efficient Building Control



Constraints

- Safety and mechanical constraints: $u_k \in \mathcal{U}_k$.
- Air quality: $\dot{V}_{sa} \geq \dot{V}_{sa,min}$.
- Thermal comfort:
  - Predicted Mean Vote (PMV) index: predicts mean of thermal comfort responses by occupants, on the scale: +3 (hot), +2 (warm), +1 (slightly warm), 0 (neutral), −1 (slightly cool), −2 (cool), −3 (cold). PMV should be close to 0.
  - Predicted Percentage Dissatisfied (PPD) index: predicted percentage of dissatisfied people. PMV and PPD has a nonlinear relation (in perfect condition $PPD(PMV = 0) = 5\%$).
  - PMV/PPD can be calculated as nonlinear functions of temperature, humidity, pressure, air velocity, etc. (cf. ASHRAE manuals).
  - Constraint on PMV/PPD gives (nonlinear) constraint on x_k.
  - Simplified as $x_k \in \mathcal{X}_k$ (convex).

Constrained Infinite Time Optimal Control

$$J_0^*(x(0)) = \min \sum_{k=0}^{\infty} q(x_k, u_k)$$

$$\text{s.t. } x_{k+1} = Ax_k + Bu_k, k = 0, \ldots, N-1$$

$$x_k \in \mathcal{X}, u_k \in \mathcal{U}, k = 0, \ldots, N-1$$

$$x_0 = x(0)$$

- **Stage cost** $q(x, u)$ describes "cost" of being in state x and applying input u
- Optimizing over a trajectory provides a **tradeoff between short- and long-term benefits** of actions
- We'll see that such a control law has many beneficial properties...
  ... but we can't compute it: there are an **infinite number of variables**

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Constrained Finite Time Optimal Control (CFTOC)

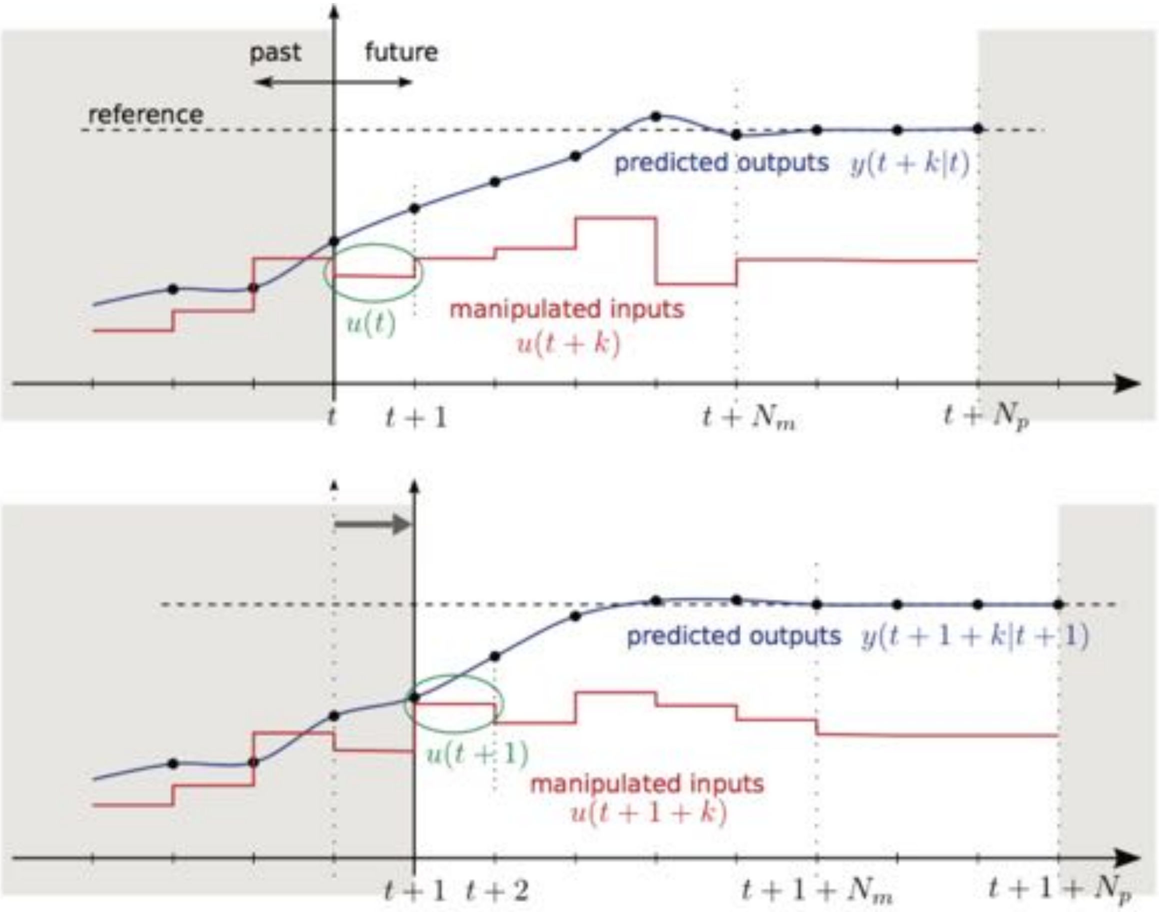
$$
\begin{aligned}
J_t^*(x(t)) = \quad \min_{U_t} \quad & p(x_{t+N}) + \sum_{k=0}^{N-1} q(x_{t+k}, u_{t+k}) \\
\text{subj. to} \quad & x_{t+k+1} = Ax_{t+k} + Bu_{t+k}, \ k = 0, \ldots, N-1 \\
& x_{t+k} \in \mathcal{X}, \ u_{t+k} \in \mathcal{U}, \ k = 0, \ldots, N-1 \\
& x_{t+N} \in \mathcal{X}_f \\
& x_t = x(t)
\end{aligned}
$$

where $\mathcal{U}_t = \{u_t, \ldots, u_{t+N-1}\}$.

Truncate after a finite horizon:

- $p(x_{t+N})$: Approximates the 'tail' of the cost
- $\mathcal{X}_f$: Approximates the 'tail' of the constraints

On-line Receding Horizon Control



1. At each sampling time, solve a **CFTOC**.
2. Apply the optimal input **only during** $[t, t+1]$
3. At $t+1$ solve a CFTOC over a **shifted horizon** based on new state measurements

On-line Receding Horizon Control

1) MEASURE the state $x(t)$ at time instance t
2) OBTAIN $U_t^*(x(t))$ by solving the optimization problem in (1)
3) IF $U_t^*(x(t)) = \emptyset$ THEN 'problem infeasible' STOP
4) APPLY the first element u_t^* of U_t^* to the system
5) WAIT for the new sampling time $t+1$, GOTO 1)

Note that we need a constrained optimization solver for step 2).

Unconstrained problem

Linear model:

$$\begin{cases} x(t+1) &= Ax(t) + Bu(t) \\ y(t) &= Cx(t) \end{cases}$$

$x \in \mathbb{R}^n$, $u \in \mathbb{R}^m$
$y \in \mathbb{R}^p$

Unconstrained problem

Linear model: $\begin{cases} x(t+1) = Ax(t) + Bu(t) \\ y(t) = Cx(t) \end{cases}$ $x \in \mathbb{R}^n$, $u \in \mathbb{R}^m$, $y \in \mathbb{R}^p$

- Goal: find $u^*(0), u^*(1), \ldots, u^*(N-1)$

$$J(x(0), U) = \sum_{k=0}^{N-1} \left[x'(k)Qx(k) + u'(k)Ru(k) \right] + x'(N)Px(N)$$

$$U = [u'(0)\ u'(1)\ \ldots\ u'(N-1)]'$$

$u^*(0), u^*(1), \ldots, u^*(N-1)$ is the input sequence that steers the state to the origin "optimally"

Unconstrained problem

$$J(x(0),U) = x'(0)Qx(0) + \begin{bmatrix} x'(1) & x'(2) & \dots & x'(N-1) & x'(N) \end{bmatrix} \overbrace{\begin{bmatrix} Q & 0 & 0 & \dots & 0 \\ 0 & Q & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & \dots & 0 & Q & 0 \\ 0 & 0 & \dots & 0 & P \end{bmatrix}}^{\bar{Q}} \cdot$$

$$\cdot \begin{bmatrix} x(1) \\ x(2) \\ \vdots \\ x(N-1) \\ x(N) \end{bmatrix} + \begin{bmatrix} u'(0) & u'(1) & \dots & u'(N-1) \end{bmatrix} \underbrace{\begin{bmatrix} R & 0 & \dots & 0 \\ 0 & R & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & \dots & 0 & R \end{bmatrix}}_{\bar{R}} \begin{bmatrix} u(0) \\ u(1) \\ \vdots \\ u(N-1) \end{bmatrix}$$

Unconstrained problem

$$\begin{bmatrix} x(1) \\ x(2) \\ \vdots \\ x(N) \end{bmatrix} = \overbrace{\begin{bmatrix} B & 0 & \dots & 0 \\ AB & B & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ A^{N-1}B & A^{N-2}B & \dots & B \end{bmatrix}}^{\bar{S}} \begin{bmatrix} u(0) \\ u(1) \\ \dots \\ u(N-1) \end{bmatrix} + \underbrace{\begin{bmatrix} A \\ A^2 \\ \vdots \\ A^N \end{bmatrix}}_{\bar{T}} x(0)$$

Unconstrained problem

$$\begin{bmatrix} x(1) \\ x(2) \\ \vdots \\ x(N) \end{bmatrix} = \overbrace{\begin{bmatrix} B & 0 & \dots & 0 \\ AB & B & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ A^{N-1}B & A^{N-2}B & \dots & B \end{bmatrix}}^{\bar{S}} \begin{bmatrix} u(0) \\ u(1) \\ \dots \\ u(N-1) \end{bmatrix} + \underbrace{\begin{bmatrix} A \\ A^2 \\ \vdots \\ A^N \end{bmatrix}}_{\bar{T}} x(0)$$

$$\begin{aligned} J(x(0), U) &= x'(0)Qx(0) + (\bar{S}U + \bar{T}x(0))'\bar{Q}(\bar{S}U + \bar{T}x(0)) + U'\bar{R}U \\ &= \frac{1}{2}U' \underbrace{2(\bar{R} + \bar{S}'\bar{Q}\bar{S})}_{H} U + x'(0) \underbrace{2\bar{T}'\bar{Q}\bar{S}}_{F} U + \frac{1}{2}x'(0) \underbrace{2(Q + \bar{T}'\bar{Q}\bar{T})}_{Y} x(0) \end{aligned}$$

Unconstrained problem

$$J(x(0), U) = \frac{1}{2}U'HU + x'(0)FU + \frac{1}{2}x'(0)Yx(0)$$

$$U = [u'(0)\ u'(1)\ \dots\ u'(N-1)]'$$

Unconstrained problem

$$J(x(0),U) = \frac{1}{2}U'HU + x'(0)FU + \frac{1}{2}x'(0)Yx(0)$$

$$U = [u'(0)\ u'(1)\ \ldots\ u'(N-1)]'$$

The optimum is obtained by zeroing the gradient

$$\nabla_U J(x(0),U) = HU + F'x(0) = 0$$

Unconstrained problem

The optimum is obtained by zeroing the gradient

$$\nabla_U J(x(0), U) = HU + F'x(0) = 0$$

and hence

$$U^* = \begin{bmatrix} u^*(0) \\ u^*(1) \\ \vdots \\ u^*(N-1) \end{bmatrix} = -H^{-1}F'x(0)$$

Alternative approach: use dynamic programming to find U^*
(Riccati iterations)

Example

Plant model:

$$x_{k+1} = \begin{bmatrix} 1.1 & 2 \\ 0 & 0.95 \end{bmatrix} x_k + \begin{bmatrix} 0 \\ 0.0787 \end{bmatrix} u_k$$

$$y_k = \begin{bmatrix} -1 & 1 \end{bmatrix} x_k$$

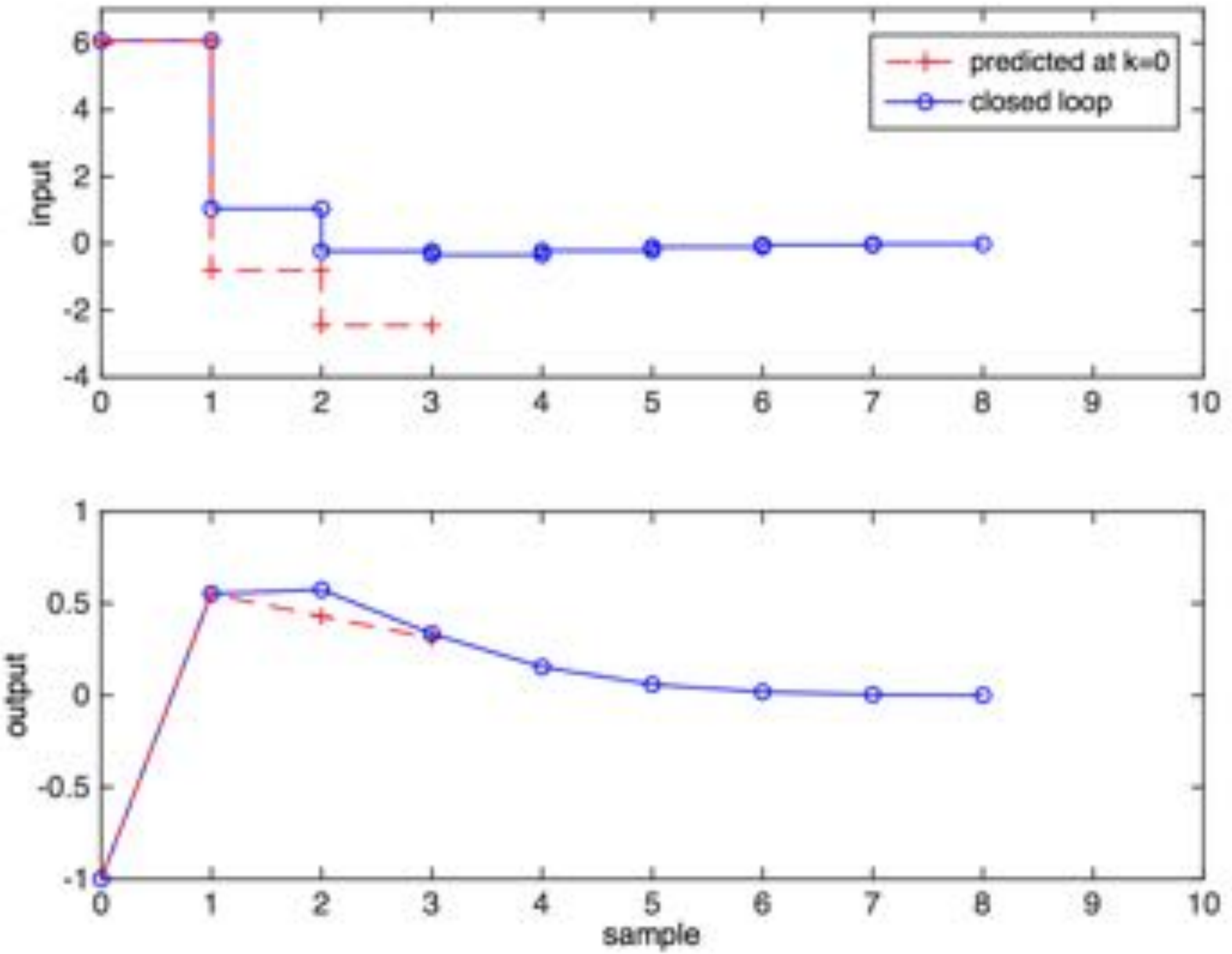
Cost:

$$\sum_{i=0}^{N-1} (y_{i|k}^2 + u_{i|k}^2) + y_{N|k}^2$$

Prediction horizon: $N = 3$

Free variables in predictions: $\mathbf{u}_k = \begin{bmatrix} u_{0|k} \\ u_{1|k} \\ u_{2|k} \end{bmatrix}$

Example



predicted at k=0
closed loop
input
6
4
2
0
-2
-4
output
1
0.5
0
-0.5
-1
0 1 2 3 4 5 6 7 8 9 10
sample

Plant model: $x_{k+1} = Ax_k + Bu_k, \quad y_k = Cx_k$

$$A = \begin{bmatrix} 1.1 & 2 \\ 0 & 0.95 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0.079 \end{bmatrix}, \quad C = \begin{bmatrix} -1 & 1 \end{bmatrix}$$

Prediction horizon $N = 4$: $$\mathcal{C} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0.079 & 0 & 0 & 0 \\ 0.157 & 0 & 0 & 0 \\ 0.075 & 0.079 & 0 & 0 \\ 0.323 & 0.157 & 0 & 0 \\ 0.071 & 0.075 & 0.079 & 0 \\ 0.497 & 0.323 & 0.157 & 0 \\ 0.068 & 0.071 & 0.075 & 0.079 \end{bmatrix}$$

Cost matrices $Q = C^T C$, $R = 0.01$, and $P = Q$:

$$H = \begin{bmatrix} 0.271 & 0.122 & 0.016 & -0.034 \\ 0.122 & 0.086 & 0.014 & -0.020 \\ 0.016 & 0.014 & 0.023 & -0.007 \\ -0.034 & -0.020 & -0.007 & 0.016 \end{bmatrix} \qquad F = \begin{bmatrix} 0.977 & 4.925 \\ 0.383 & 2.174 \\ 0.016 & 0.219 \\ -0.115 & -0.618 \end{bmatrix}$$

$$G = \begin{bmatrix} 7.589 & 22.78 \\ 22.78 & 103.7 \end{bmatrix}$$

Example

Model: A, B, C as before, cost: $J_k = \sum_{i=0}^{N-1} (y_{i|k}^2 + 0.01 u_{i|k}^2) + y_{N|k}^2$

- For $N = 4$: $\mathbf{u}_k^* = -H^{-1} F x_k = \begin{bmatrix} -4.36 & -18.7 \\ 1.64 & 1.24 \\ 1.41 & 3.00 \\ 0.59 & 1.83 \end{bmatrix} x_k$

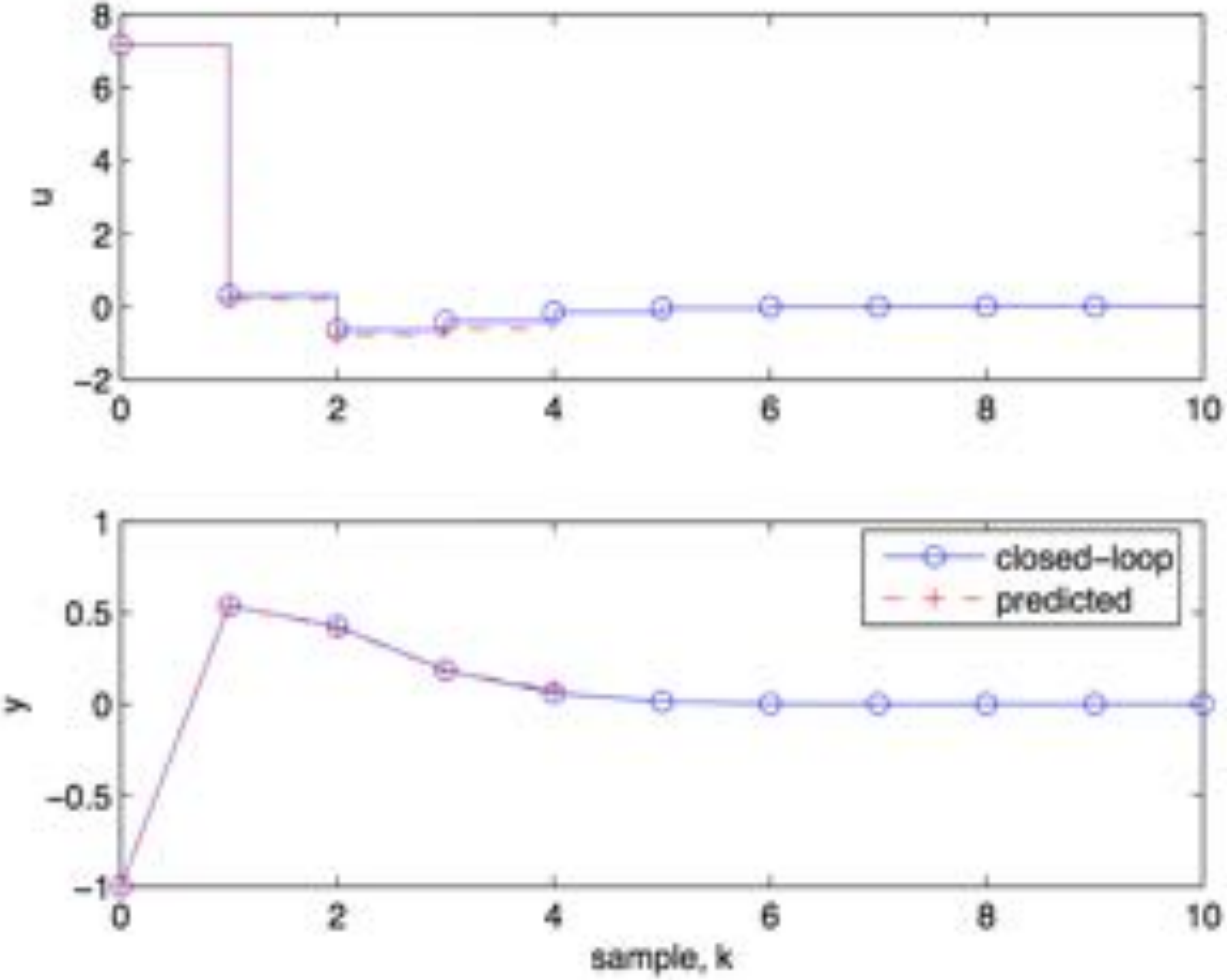
$$u_k = \begin{bmatrix} -4.36 & -18.7 \end{bmatrix} x_k$$

Example

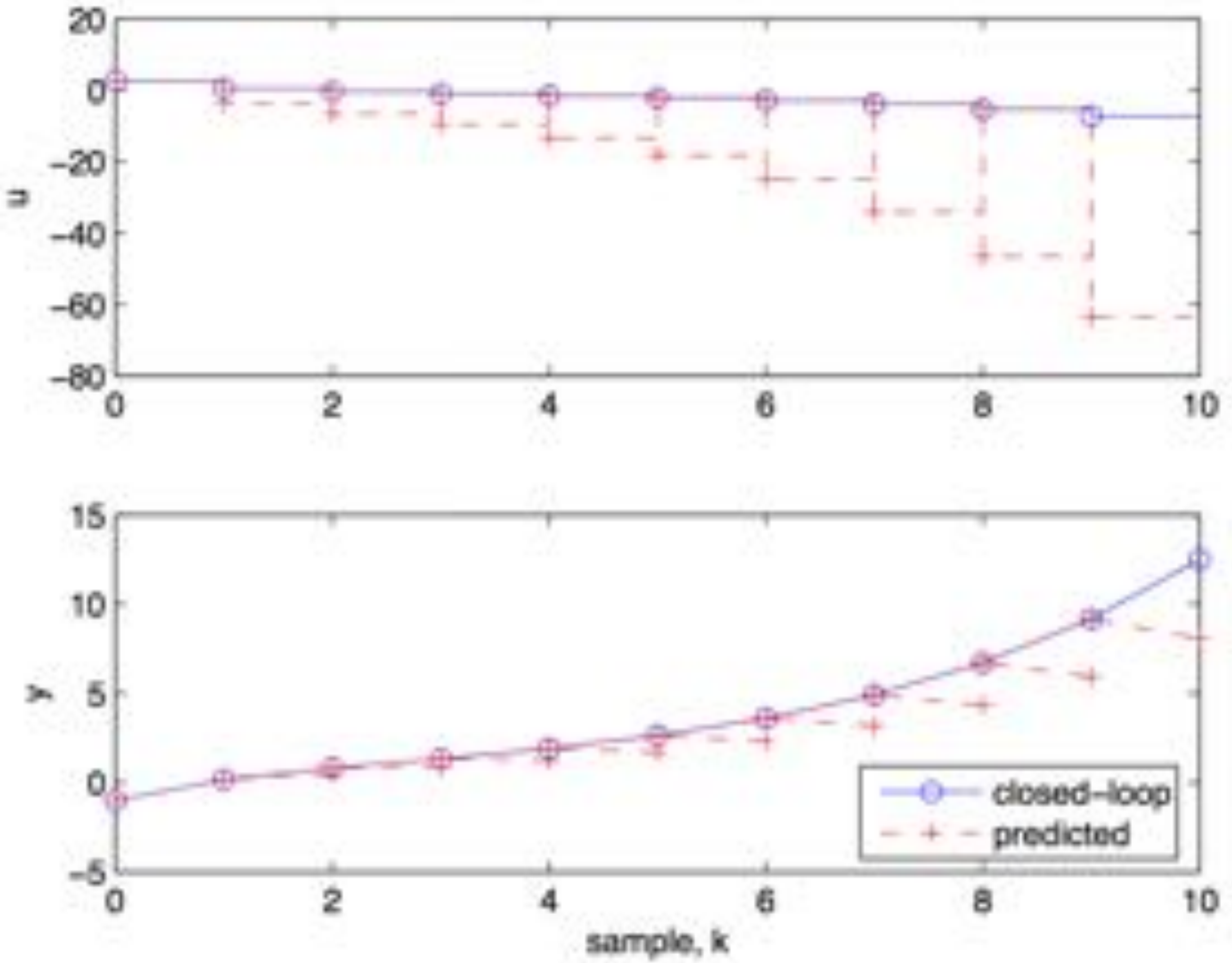
- For general N: $u_k = K_N x_k$

	$N=4$	$N=3$	$N=2$	$N=1$
K_N	$[-4.36 \quad -18.69]$	$[-3.80 \quad -16.98]$	$[1.22 \quad -3.95]$	$[5.35 \quad 5.10]$
$\lambda(A+BK_N)$	$0.29 \pm 0.17j$	$0.36 \pm 0.22j$	$1.36, 0.38$	$2.15, 0.30$
	stable	stable	unstable	unstable

Horizon: $N = 4$, $x_0 = (0.5, -0.5)$



Horizon: $N = 2$, $x_0 = (0.5, -0.5)$



Observation: predicted and closed loop responses are different for small N

MPC challenges

- *Implementation*
  MPC problem has to be solved in real-time, i.e. within the sampling interval of the system, and with available hardware (storage, processor,...).
- *Stability*
  Closed-loop stability, i.e. convergence, is not automatically guaranteed
- *Robustness*
  The closed-loop system is not necessarily robust against uncertainties or disturbances
- *Feasibility*
  Optimization problem may become infeasible at some future time step, i.e. there may not exist a plan satisfying all constraints

Literature

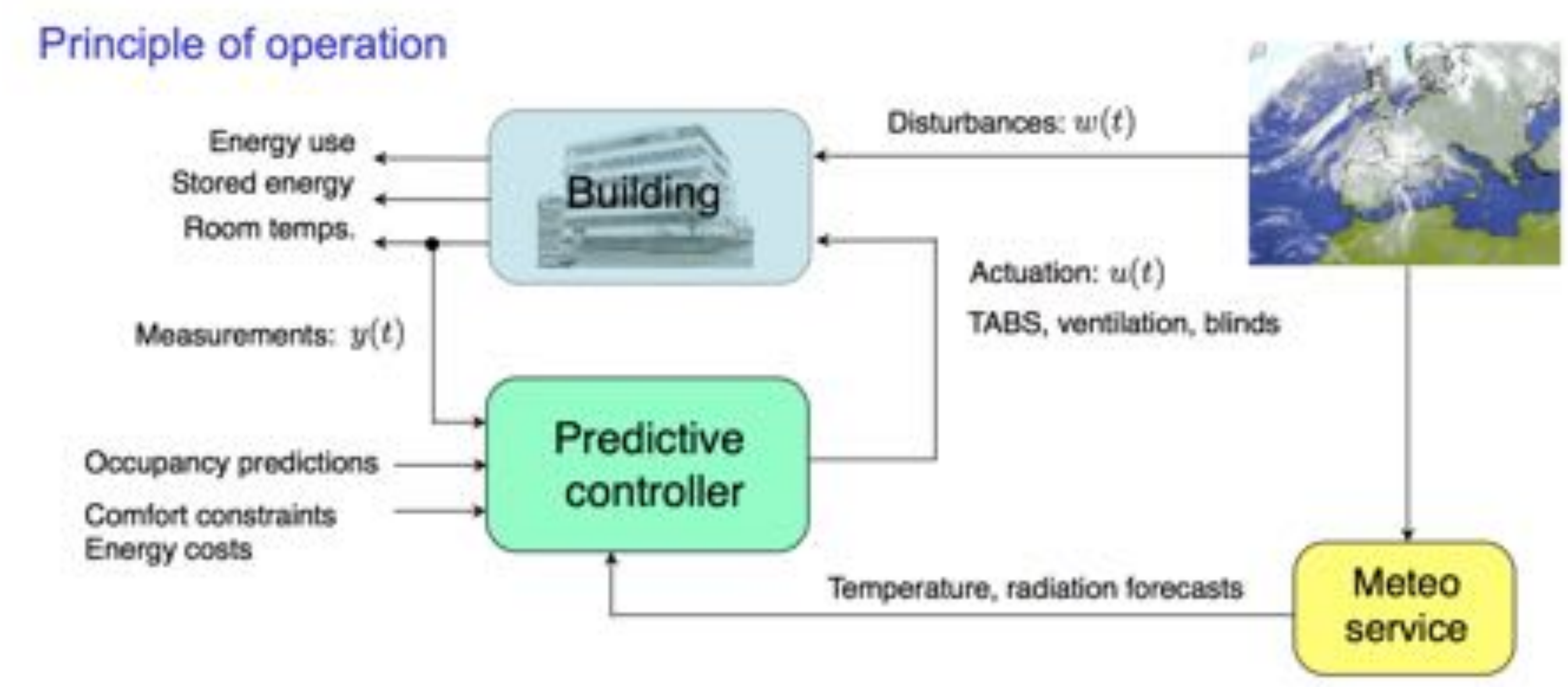
Model Predictive Control:

- Predictive Control for linear and hybrid systems, F. Borrelli, A. Bemporad, M. Morari, 2013 Cambridge University Press [http://www.mpc.berkeley.edu/mpc-course-material]
- Model Predictive Control: Theory and Design, James B. Rawlings and David Q. Mayne, 2009 Nob Hill Publishing
- Predictive Control with Constraints, Jan Maciejowski, 2000 Prentice Hall

Optimization:

- Convex Optimization, Stephen Boyd and Lieven Vandenberghe, 2004 Cambridge University Press
- Numerical Optimization, Jorge Nocedal and Stephen Wright, 2006 Springer

MPC for buildings



MPC for buildings

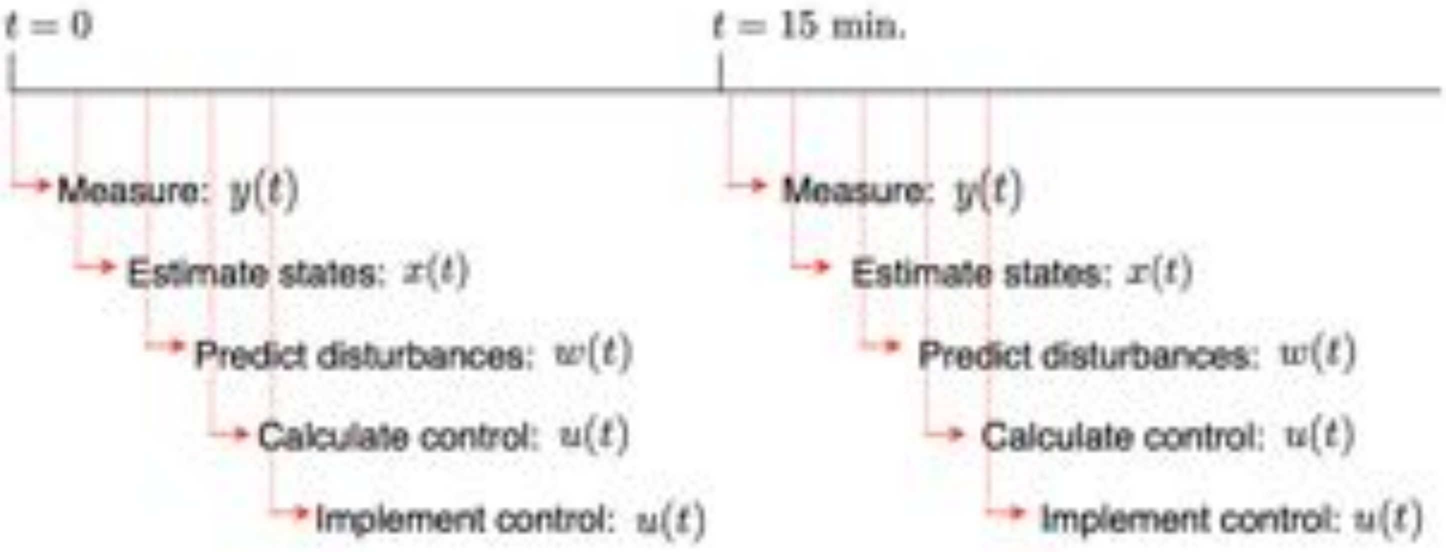
$$\text{Predicted Cost} = \underset{u(t)}{\text{minimize}} \;\; \text{Expected}\left(\sum_{t}^{t+N} \text{energy cost(t)}\right)$$ ← Minimize the predicted energy cost

$$\text{subject to} \quad u(t) \in \mathcal{U}$$ ← Actuation within limits

$$x(t) \in \mathcal{X}$$ ← Predicted temperatures within limits

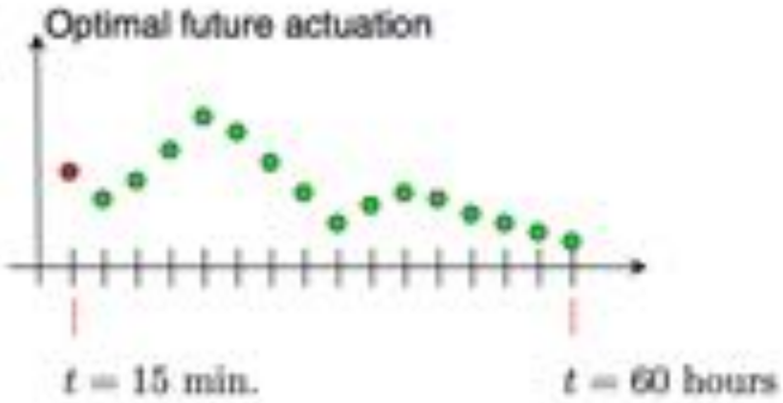
$$x(t+1) = f(x(t), u(t), w(t))$$ ← Predicted dynamics of the building

MPC controller operation

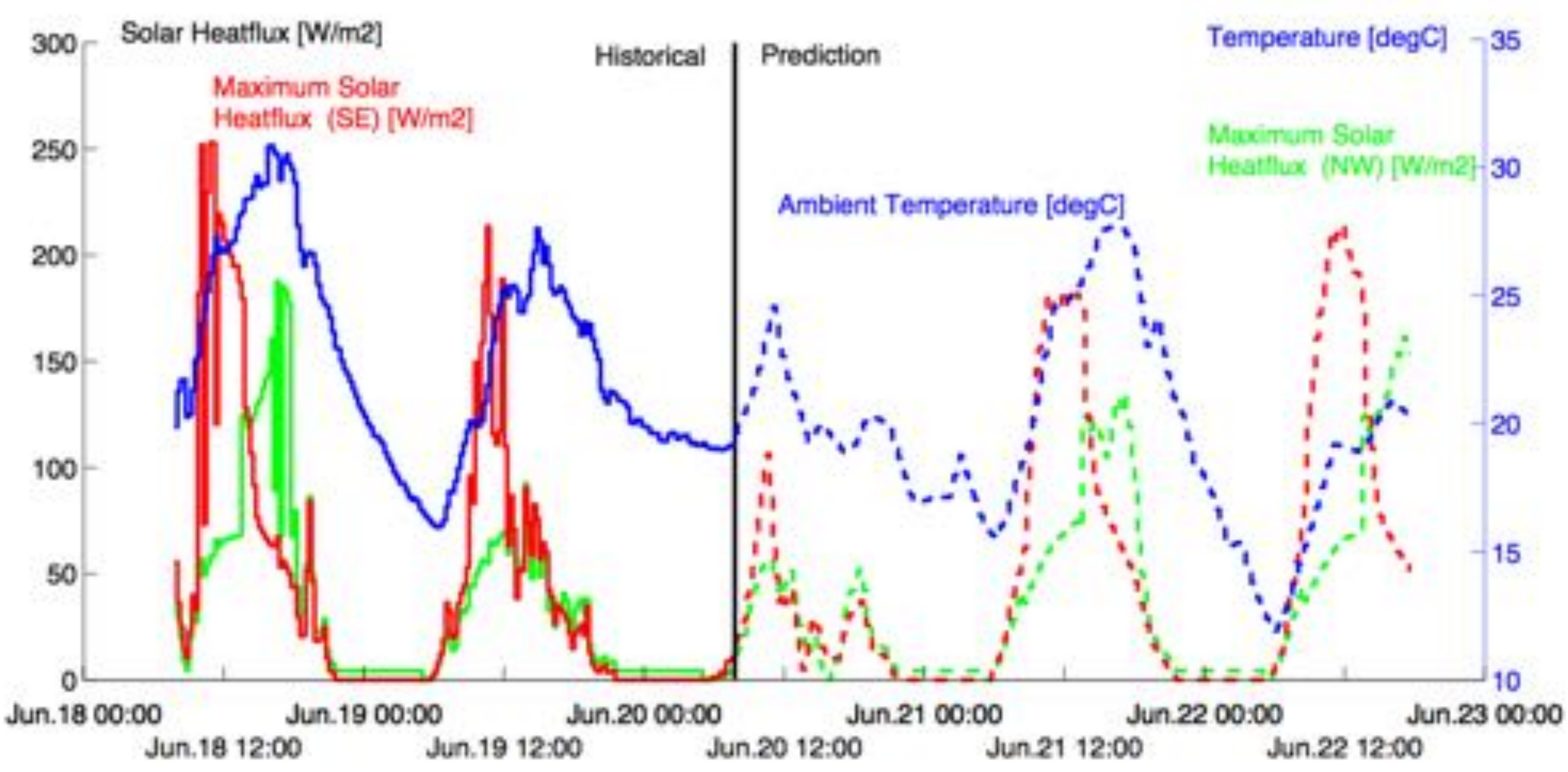


Weather forecast: 72 hours, updated every 12 hours

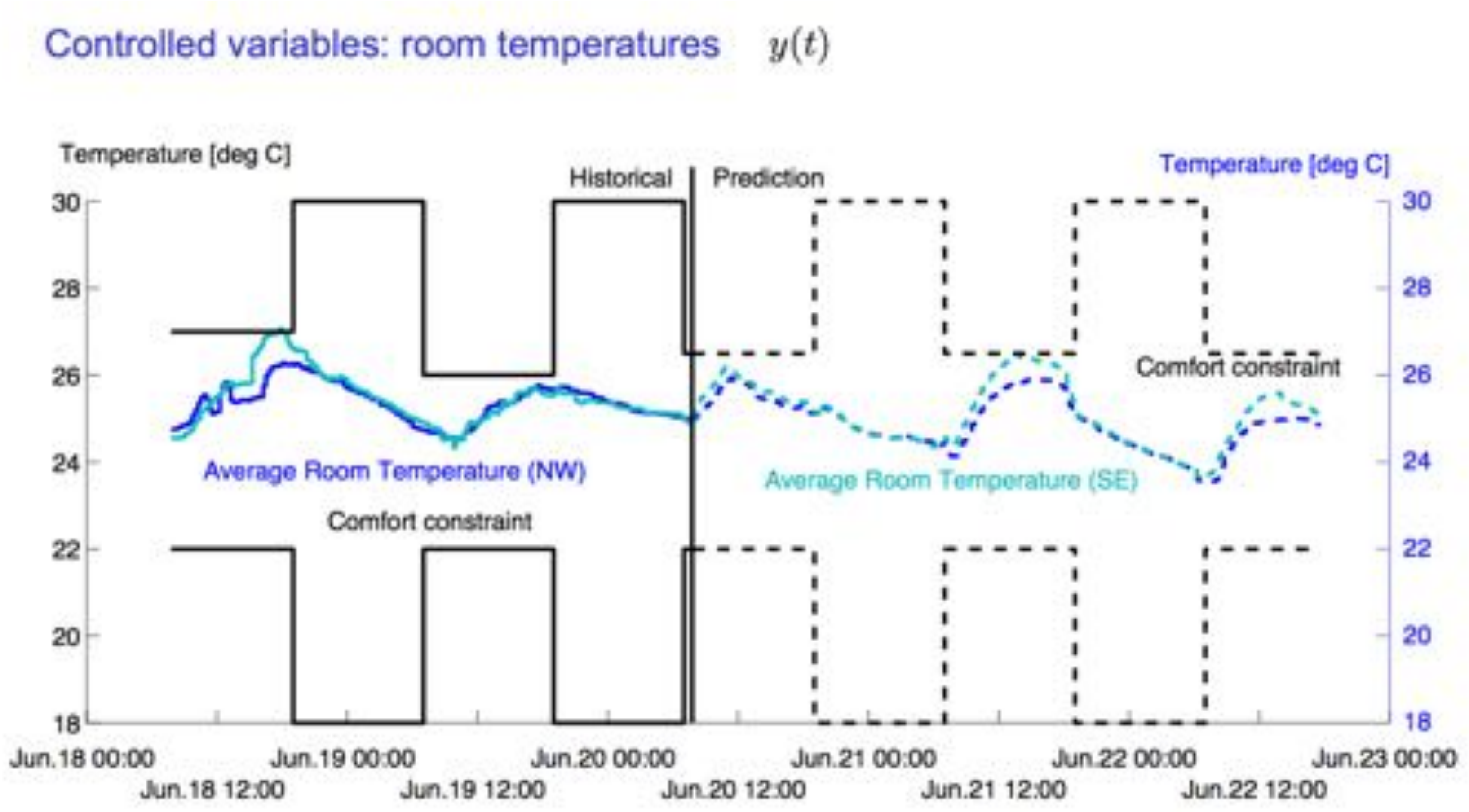
Prediction horizon: 60 hours (240 time steps ahead)



Disturbances

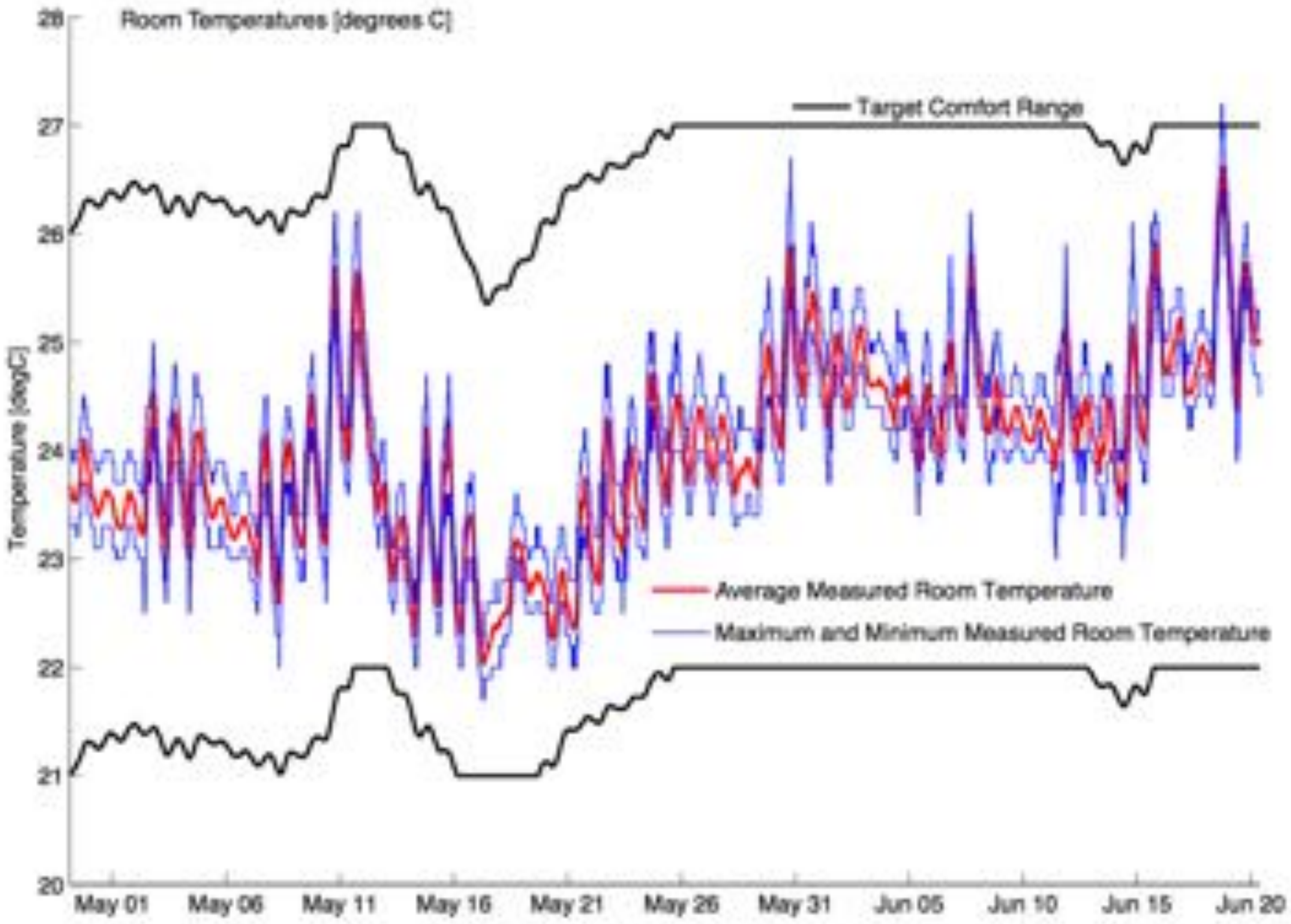


Controlled variables



Performance: room temperatures (50 days)

TABS heating was required on 18 May.

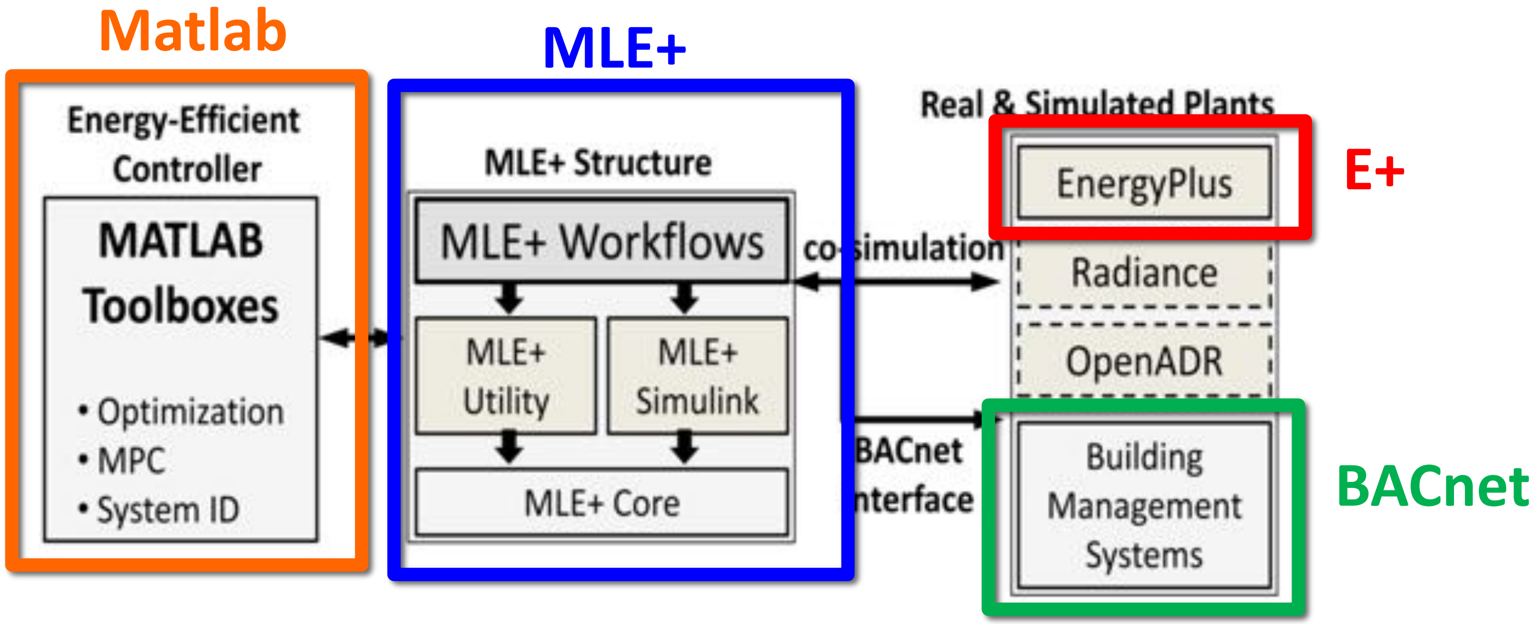


MLE+ Overview

1. High-Fidelity Physical models of the whole-building Energy Simulator **EnergyPlus**.

2. The scientific computational capability of **Matlab/Simulink**:
   I. Matlab Toolboxes
   II. Matlab Built-in Functions.

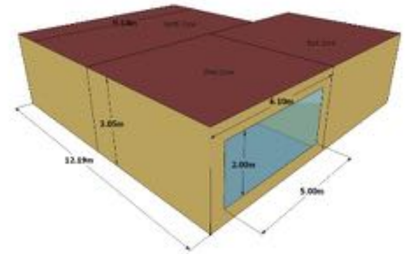
3. Control Synthesis - Building Control Deployment.



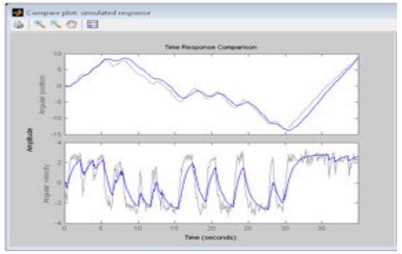
MLE+ Workflow

From Control/Scheduling Algorithms
to Synthesis and Deployment in Real Buildings

1 **EnergyPlus Building Model**



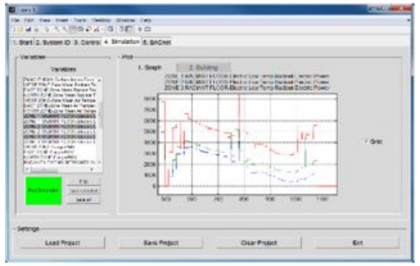
2 **System Identification**



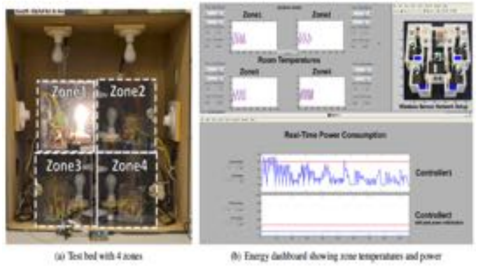
3 **Control Design in Matlab**

```
if Zone_West.Solar > 100
    % DEPLOYED WHEN SOLAR RADIATION EXCEEDS THRESHOLD
    ShadeStatus = userdata.Shade.Status.Exterior_Blind_On;
    ShadeAngle = IncidentAngle;
else
    % SHADES NOT DEPLOYED
    ShadeStatus = userdata.Shade.Status.Off;
    ShadeAngle = IncidentAngle;
end
% FEEDBACK
eplus_in_curr.ShadeStatus = ShadeStatus;
eplus_in_curr.ShadeAngle = ShadeAngle;
end
```

4 **Simulation Results**



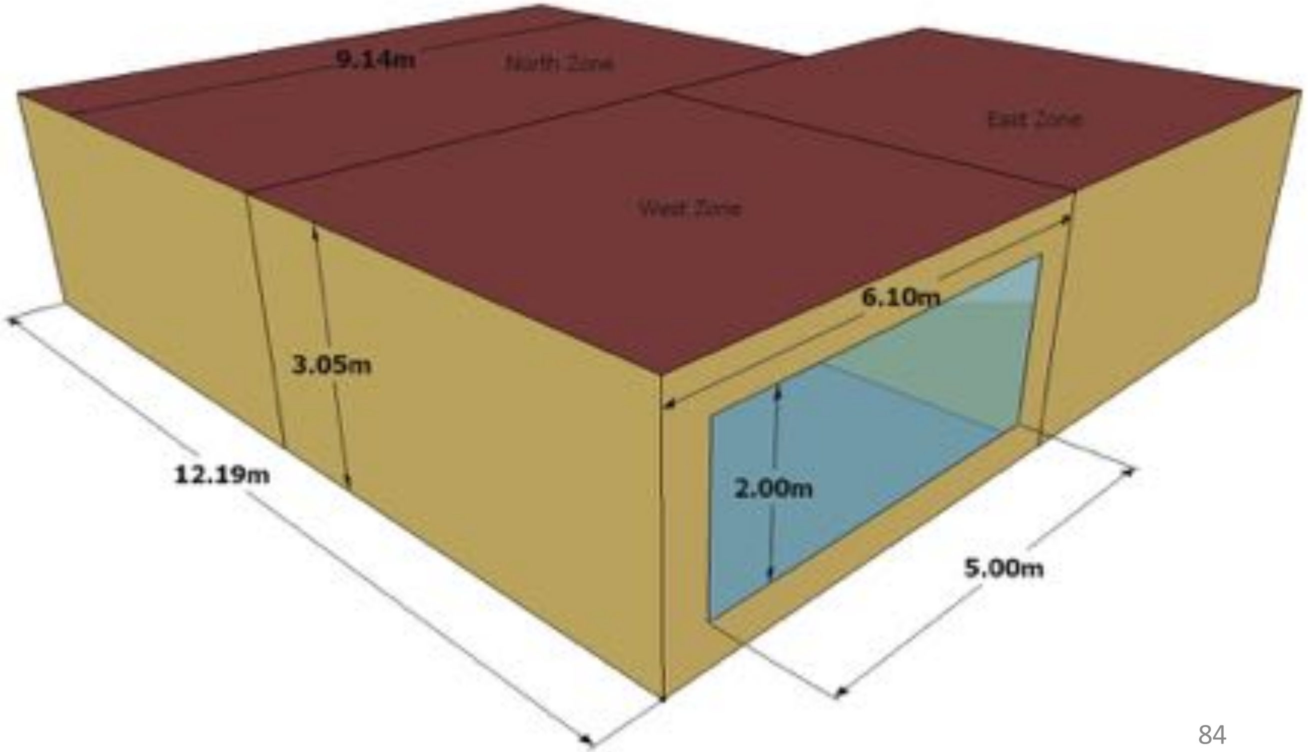
5 **Control Deployment**



Advanced Controls: **Model Predictive Control (**MPC)

EnergyPlus Building Model

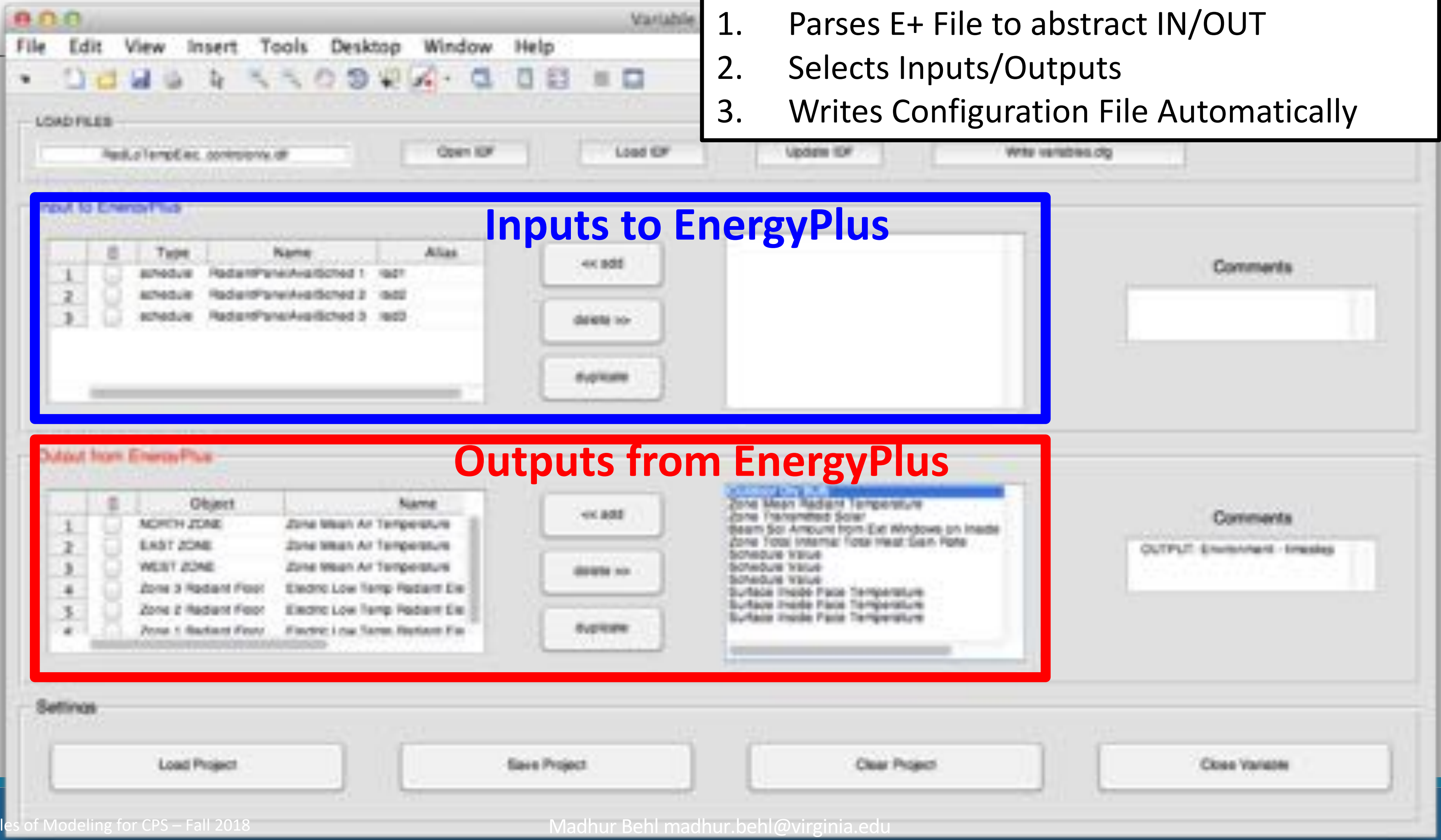
- ✓ Small office building with 3 zones
- ✓ Chicago weather file during winter
- ✓ Model Predictive Control:
  - Minimize the power consumption of the radiant heater
  - Maintain thermal comfort (22°C - 24°C)



Advanced Controls: Variable Configuration

Variable Configuration Screen

1. Parses E+ File to abstract IN/OUT
2. Selects Inputs/Outputs
3. Writes Configuration File Automatically



Advanced Controls: Input/Output Configuration

Configuration File
1. **.xml** file contains Co-Simulation Exchange Variables
2. Inputs to E+
   - Power of radiant heating system
3. Outputs from E+
   - Room temperatures
   - Radiant heating system power

```
<?xml version="1.0" encoding=

<!DOCTYPE BCVTB-variables
  SYSTEM "variables.dtd">
<BCVTB-variables><!--INPUT-->
  <variable source="Ptolemy"
    <EnergyPlus schedule="R
  </variable>
  <variable source="Ptolemy"
    <EnergyPlus schedule="R
  </variable>
  <variable source="Ptolemy">
    <EnergyPlus schedule="RadiantPanelAvailSched 3"/>
  </variable><!--OUTPUT-->
  <variable source="EnergyPlus">
    <EnergyPlus name="NORTH ZONE" type="Zone Mean Air Temperature"/>
  </variable>
  <variable source="EnergyPlus">
    <EnergyPlus name="EAST ZONE" type="Zone Mean Air Temperature"/>
  </variable>
  <variable source="EnergyPlus">
    <EnergyPlus name="WEST ZONE" type="Zone Mean Air Temperature"/>
  </variable>
  <variable source="EnergyPlus">
    <EnergyPlus name="Zone 3 Radiant Floor" type="Electric Low Temp Radiant Electric Power"/>
  </variable>
  <variable source="EnergyPlus">
    <EnergyPlus name="Zone 2 Radiant Floor" type="Electric Low Temp Radiant Electric Power"/>
  </variable>
  <variable source="EnergyPlus">
    <EnergyPlus name="Zone 1 Radiant Floor" type="Electric Low Temp Radiant Electric Power"/>
  </variable>
</BCVTB-variables>
```

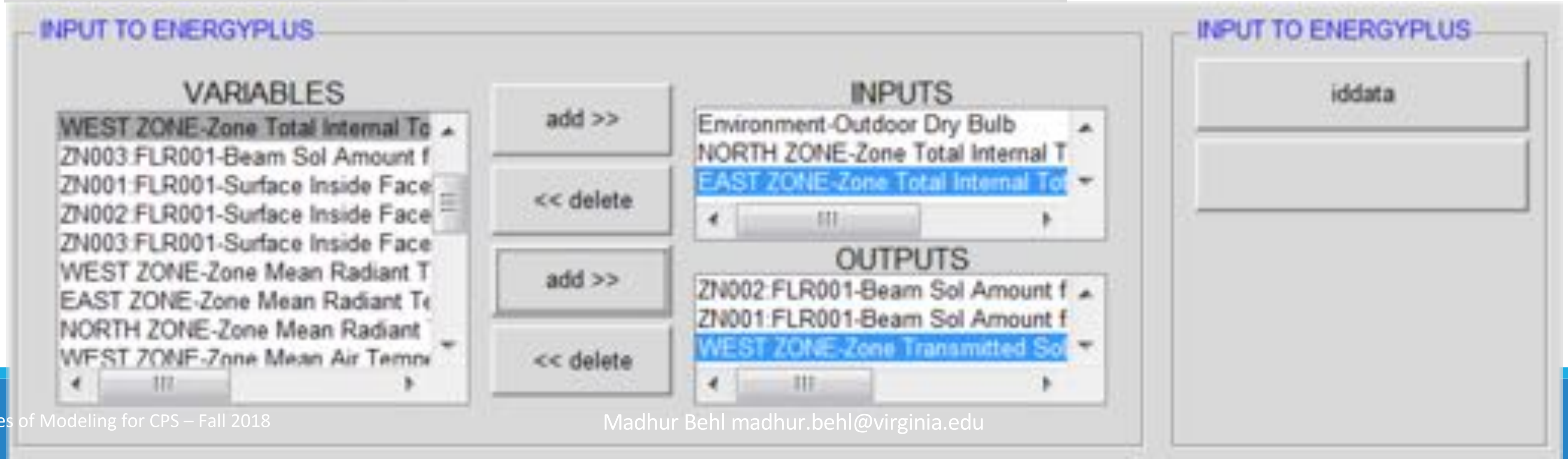
Inputs to EnergyPlus

Outputs from EnergyPlus

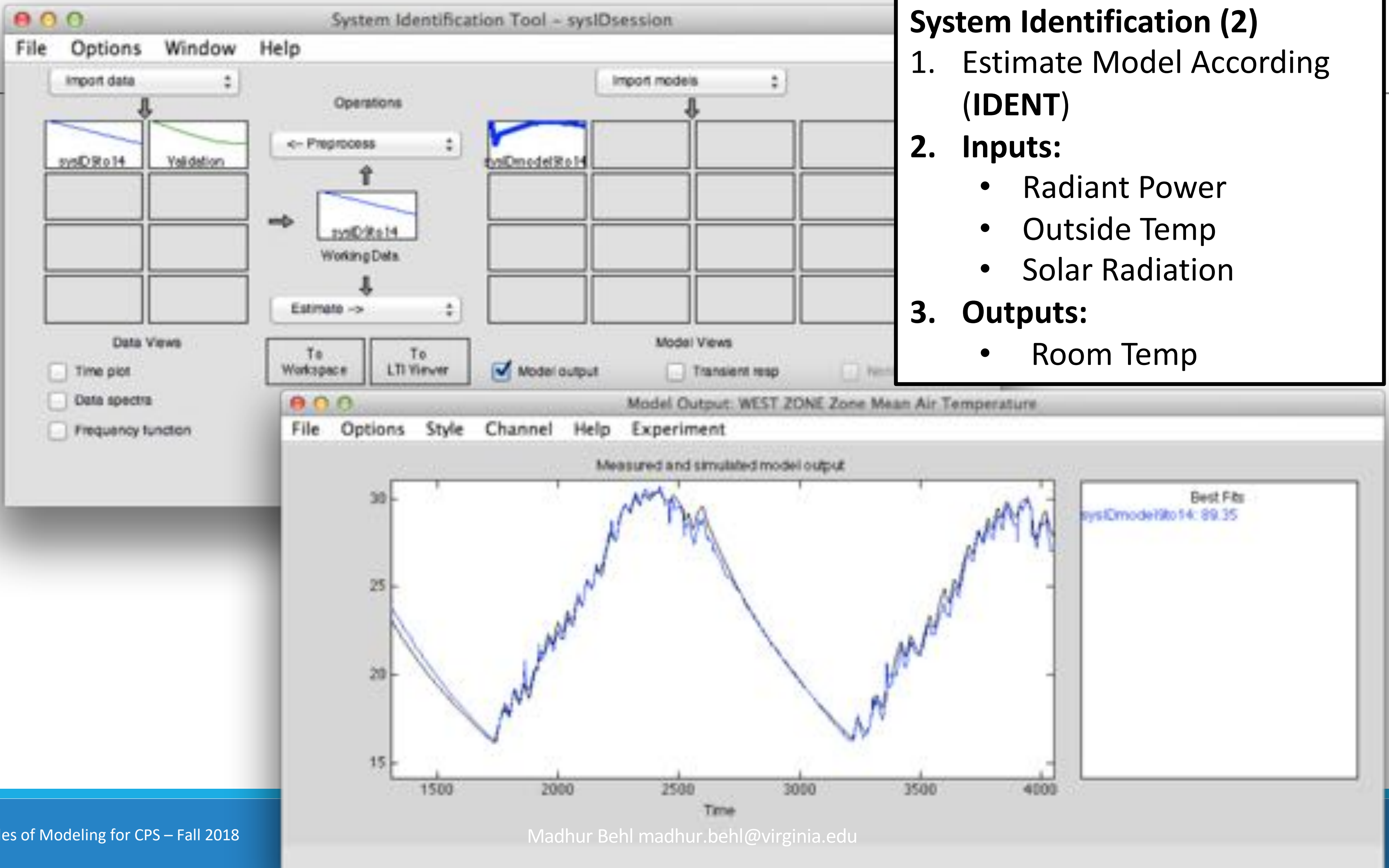
Advanced Controls: System Identification

System Identification

1. Random Signal Generator
2. **IDINPUT** (Matlab Built-in function)
3. Package Data: **IDDATA** (Matlab Built-in type)
4. Import to System Identification Toolbox (**IDENT**)



Advanced Controls: System Identification (2)



System Identification (2)

1. Estimate Model According (**IDENT**)
2. **Inputs:**
   - Radiant Power
   - Outside Temp
   - Solar Radiation
3. **Outputs:**
   - Room Temp

Advanced Controls: Control Design

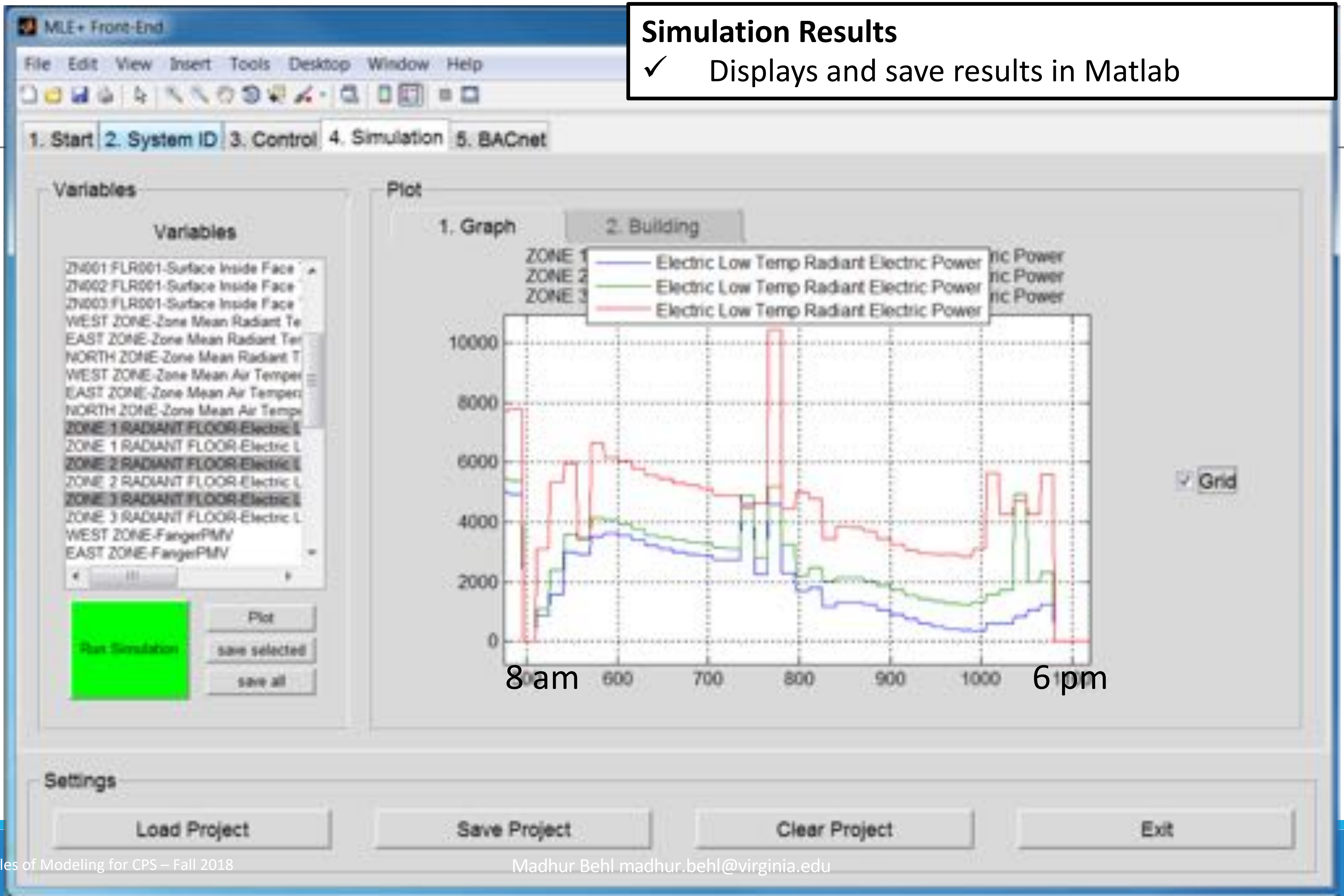
```
% GENERATE INPUT MPC
if mod(stepNumber,userdata.Ts) == 1
    [input Info] = mpcmove(userdata.mpcobj,userdata.x,y,userdata.r,userdata.v);
    input = input';
    userdata.input = input;
    % TRANSFORM POWER TO SET POINT
    % WEST - EAST - NORTH
    tsp = (y+userdata.input.*userdata.range./userdata.maxPow)-userdata.range/2;
    userdata.tsp(stepNumber,:) = tsp;
    userdata.cost(stepNumber) = Info.Cost;
    userdata.slack(stepNumber) = Info.Slack;
    if strcmp(Info.QPCode,'infeasible')
        disp('infeasible');
    end
```

- ✓ Use template script to specify controller
- ✓ Easily integrate with Matlab's Model Predictive Control toolbox.
- ✓ MPC:
  - ✓ Prediction Horizon: 2
  - ✓ Control Horizon: 9
  - ✓ Minimize Total Power Consumption

Advanced Controls: Simulation Results

Simulation Results

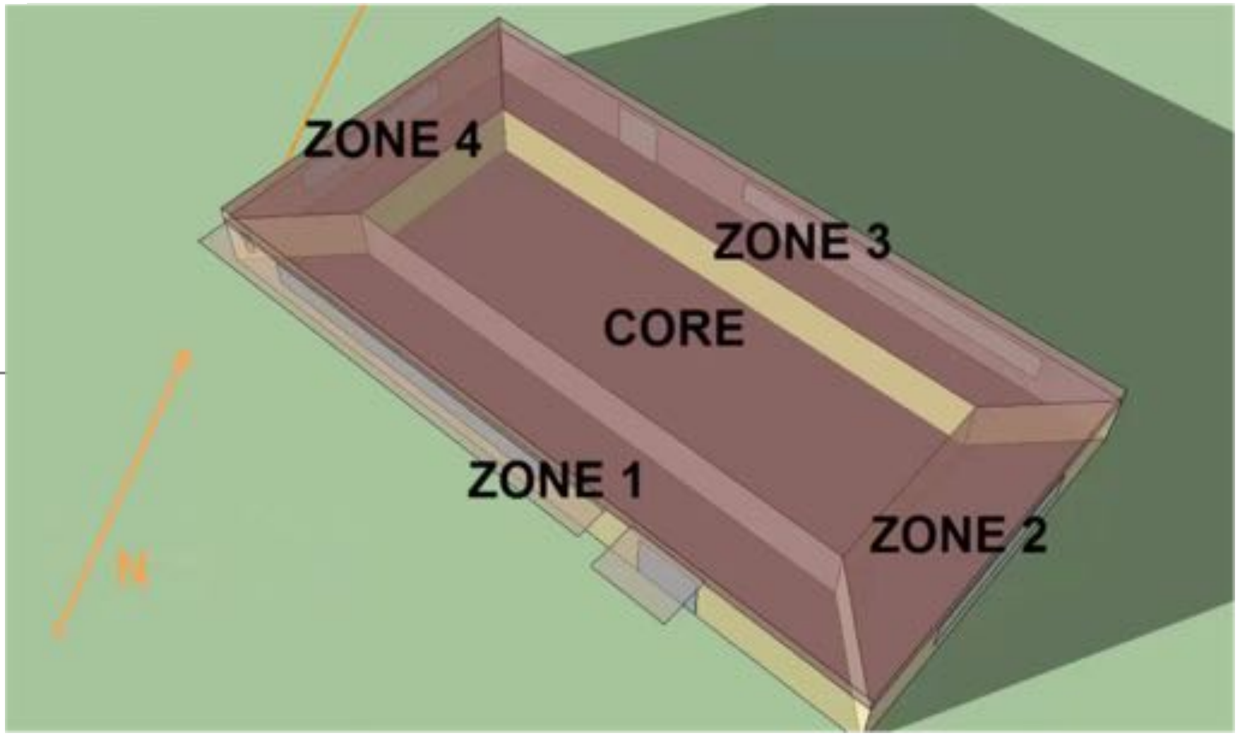
✓ Displays and save results in Matlab



Integrated Modeling: MLE+ Simulink

Simulink Example: Co-Design & Controls

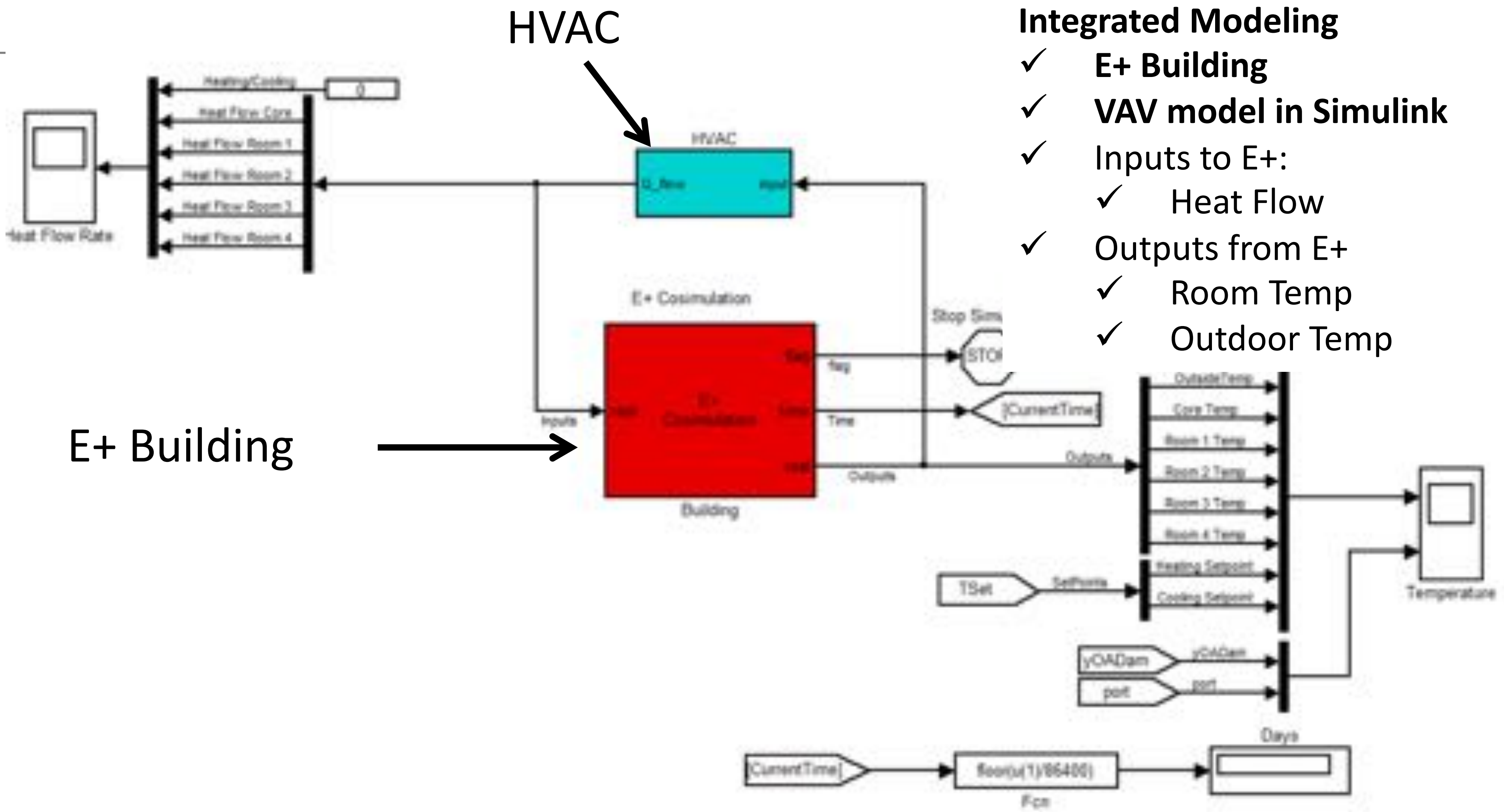
- ✓ 5 Zone Building
- ✓ California Weather File
- ✓ July 1st – 7th (Summer Time)
- ✓ VAV System



Integrated Modeling: Simulation Overview

Integrated Modeling

- ✓ **E+ Building**
- ✓ **VAV model in Simulink**
- ✓ Inputs to E+:
  - ✓ Heat Flow
- ✓ Outputs from E+
  - ✓ Room Temp
  - ✓ Outdoor Temp



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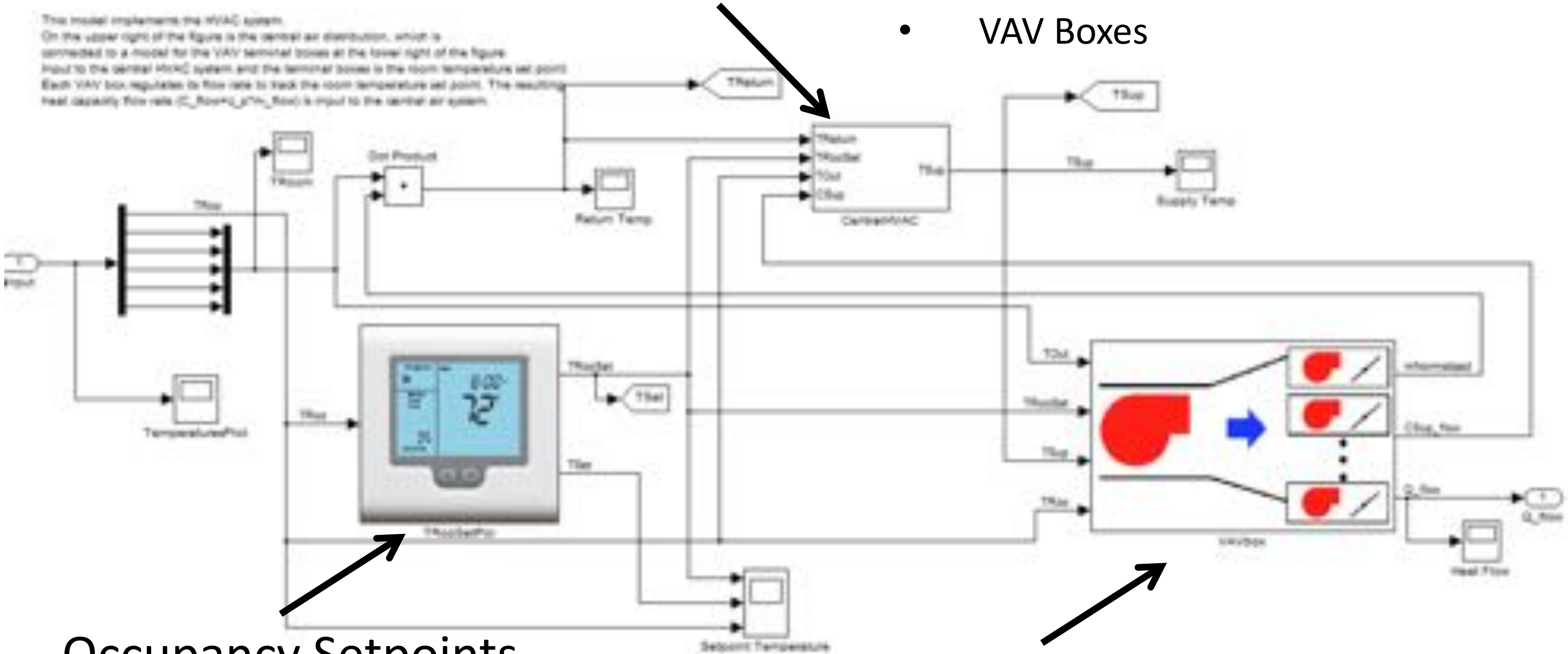
Integrated Modeling: HVAC System

HVAC System

- Temperature Setpoints according to Schedule
- Central HVAC Model
- VAV Boxes

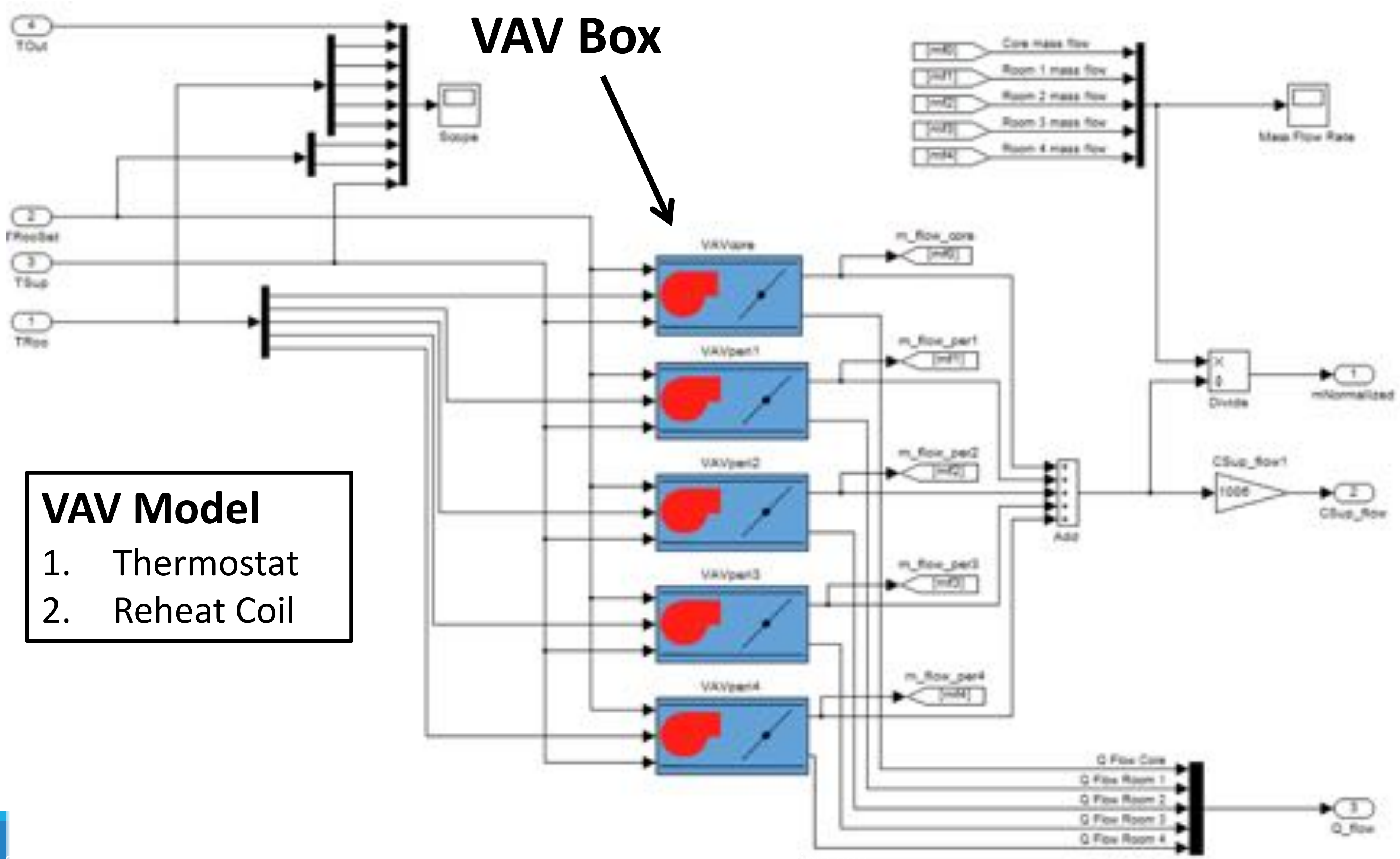
CentralHVAC



Occupancy Setpoints

VAV Boxes

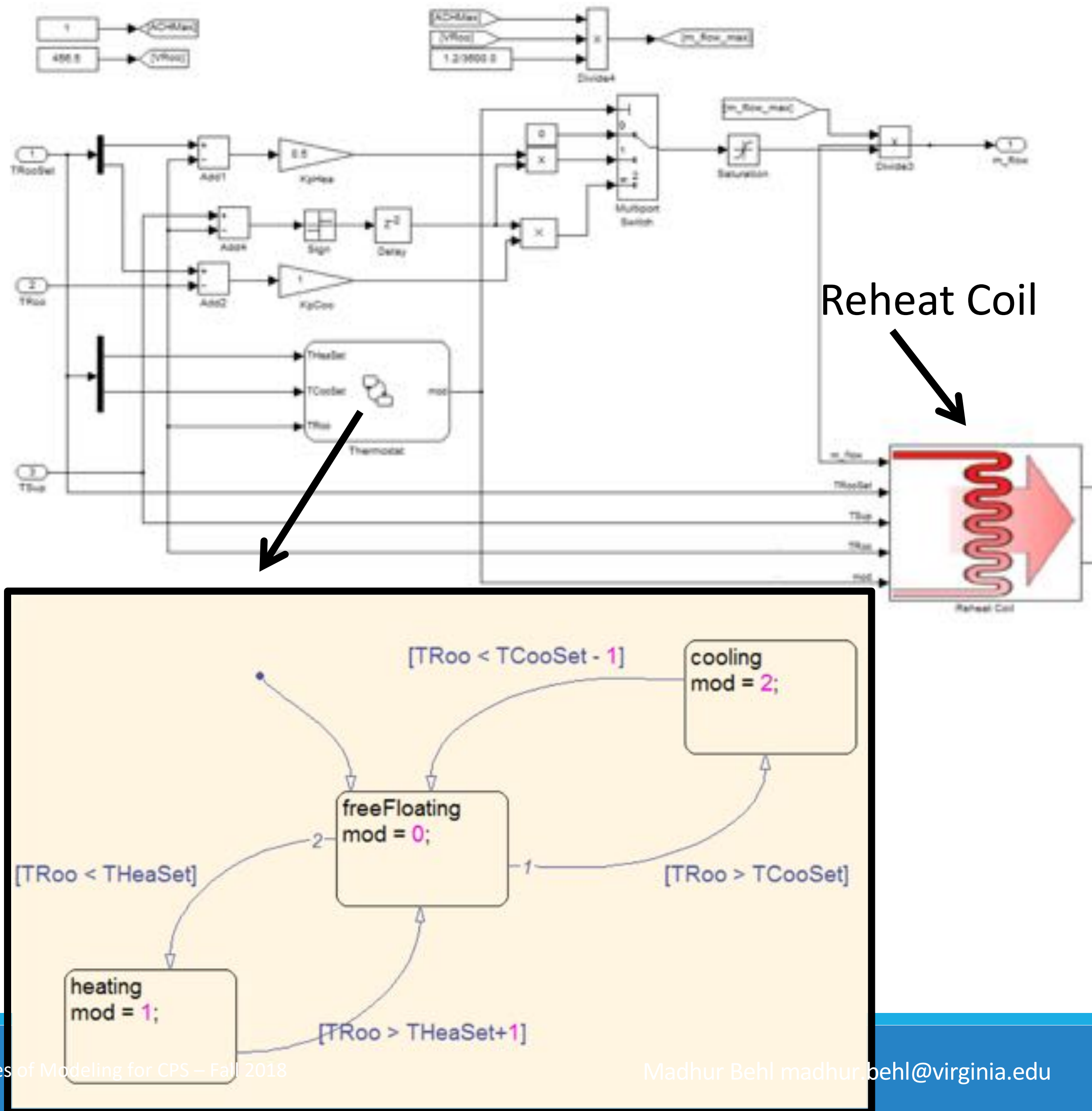
Integrated Modeling: VAV boxes



VAV Model

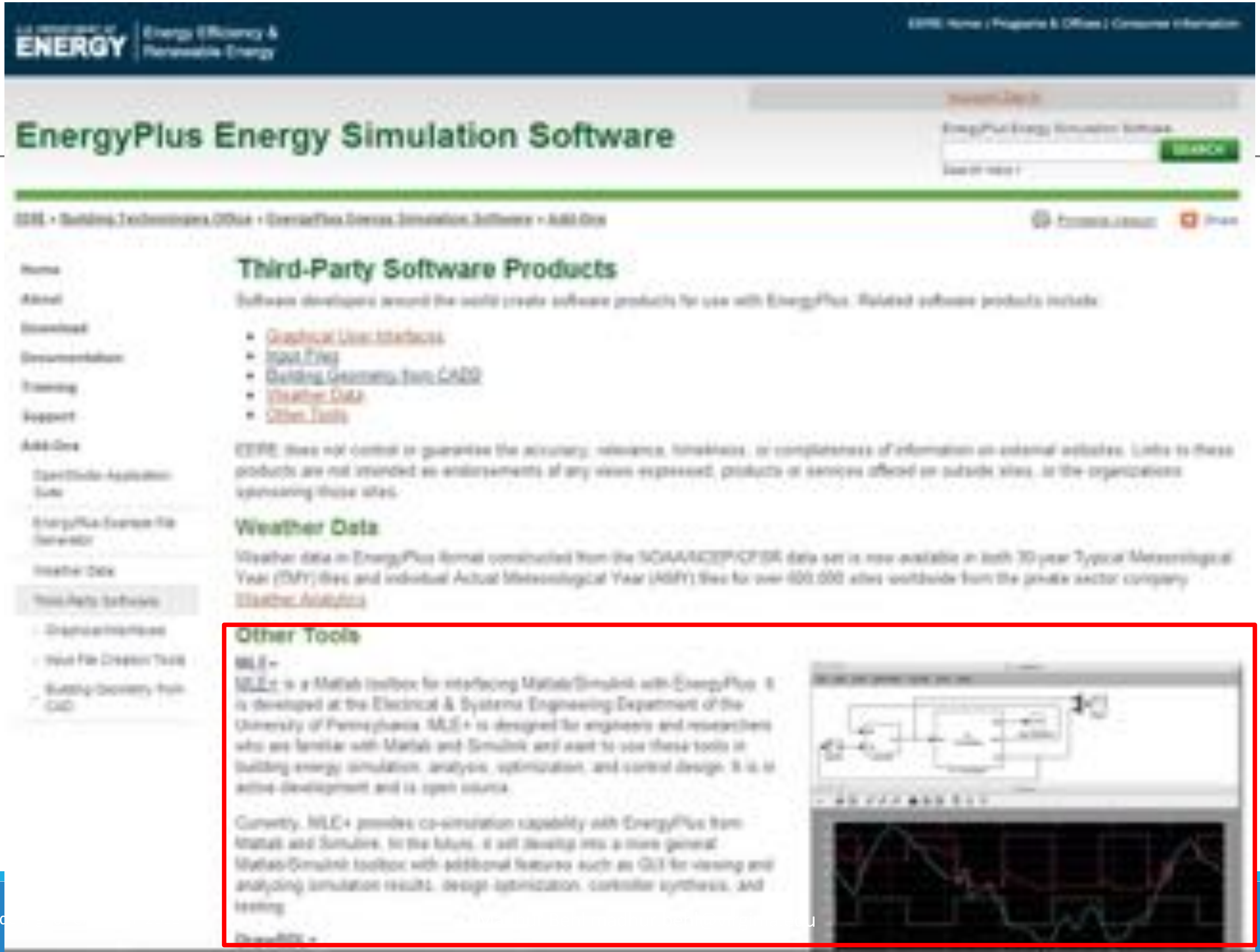
1. Thermostat
2. Reheat Coil

Integrated Modeling: VAV Box

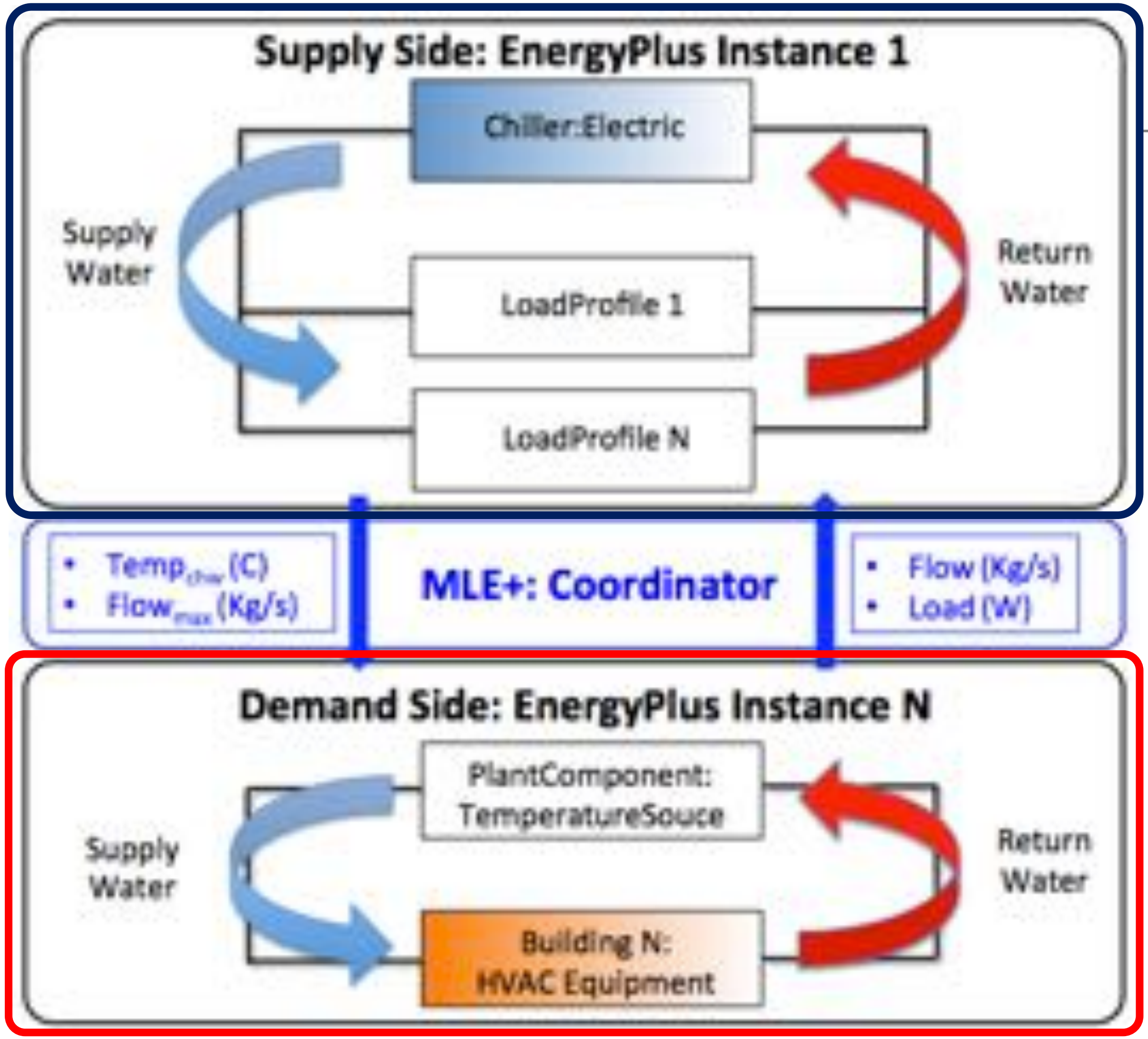


- ✓ **VAV Box**
- ✓ Inputs:
  - ✓ Setpoints
  - ✓ Room Temp
  - ✓ Supply Air Temp
- ✓ Outputs:
  - ✓ Mass Flow Rate
- ✓ Thermostat Room Level
  - ✓ Heat/FreeCool/MechanicalCool
- ✓ Reheat Coil

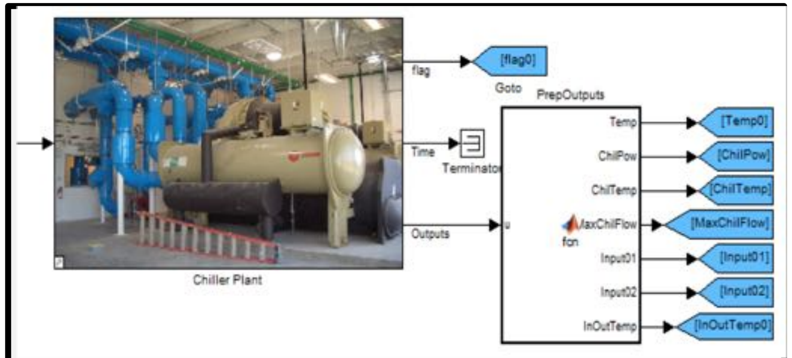
MLE+ is a featured third party tool recognized by DoE



Campus-Wide Simulation



Supply Side
EPlus **Load Profiler** object



MLE+ S function block in Simulink

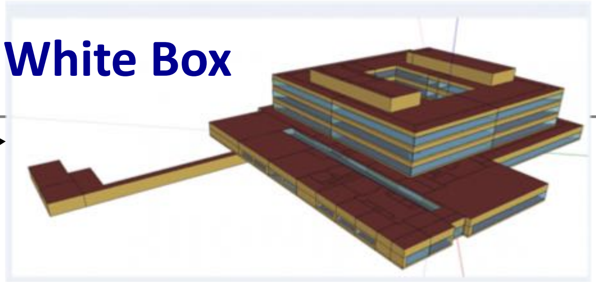
Demand Side
EPlus **TemperatureSource** object

MLE+

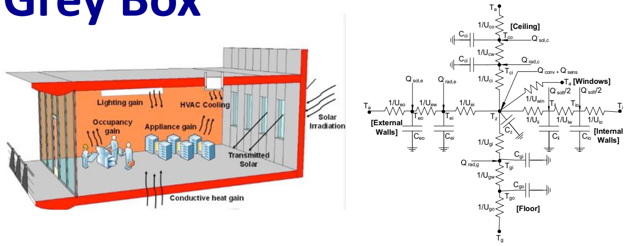
Over 400+ users

How are building models obtained today ?

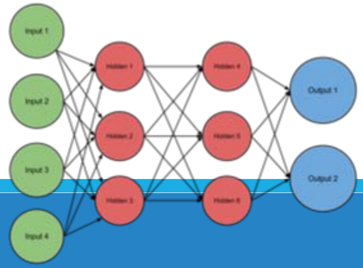
White Box



Grey Box



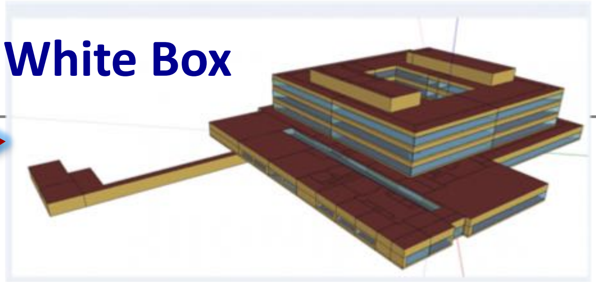
Black Box



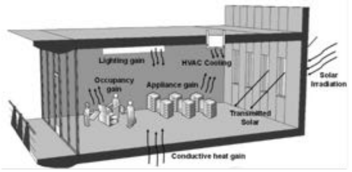
Madhur Behl madhur.behl@virginia.edu

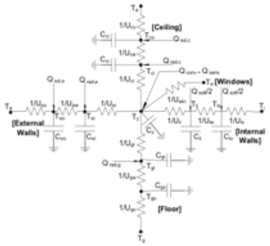
How are building models obtained today ?

White Box

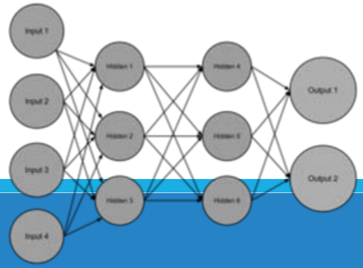


Grey Box



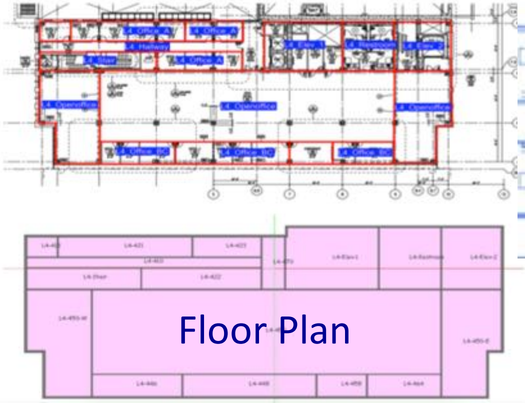


Black Box

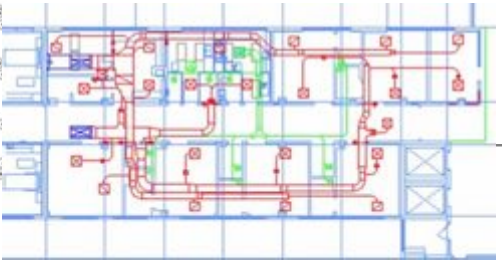


White-Box Modeling

Not always available



Floor Plan



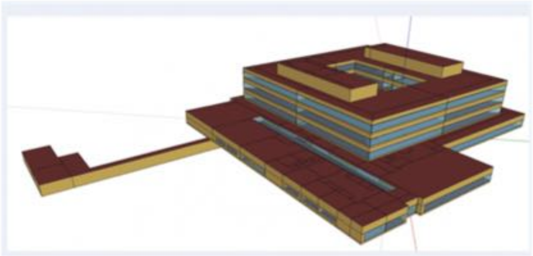
HVAC layout

Model Tuning
Data
Sensor retrofits

Set parameters
Floor by floor
Zone by zone
Wall by wall
Layer by layer
Equipment by equipment
Transfer geometry

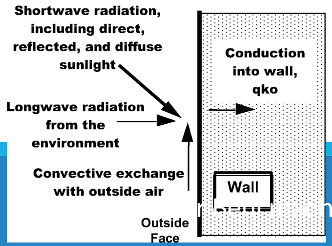
Guess nominal parameter values

Hire building modeling experts



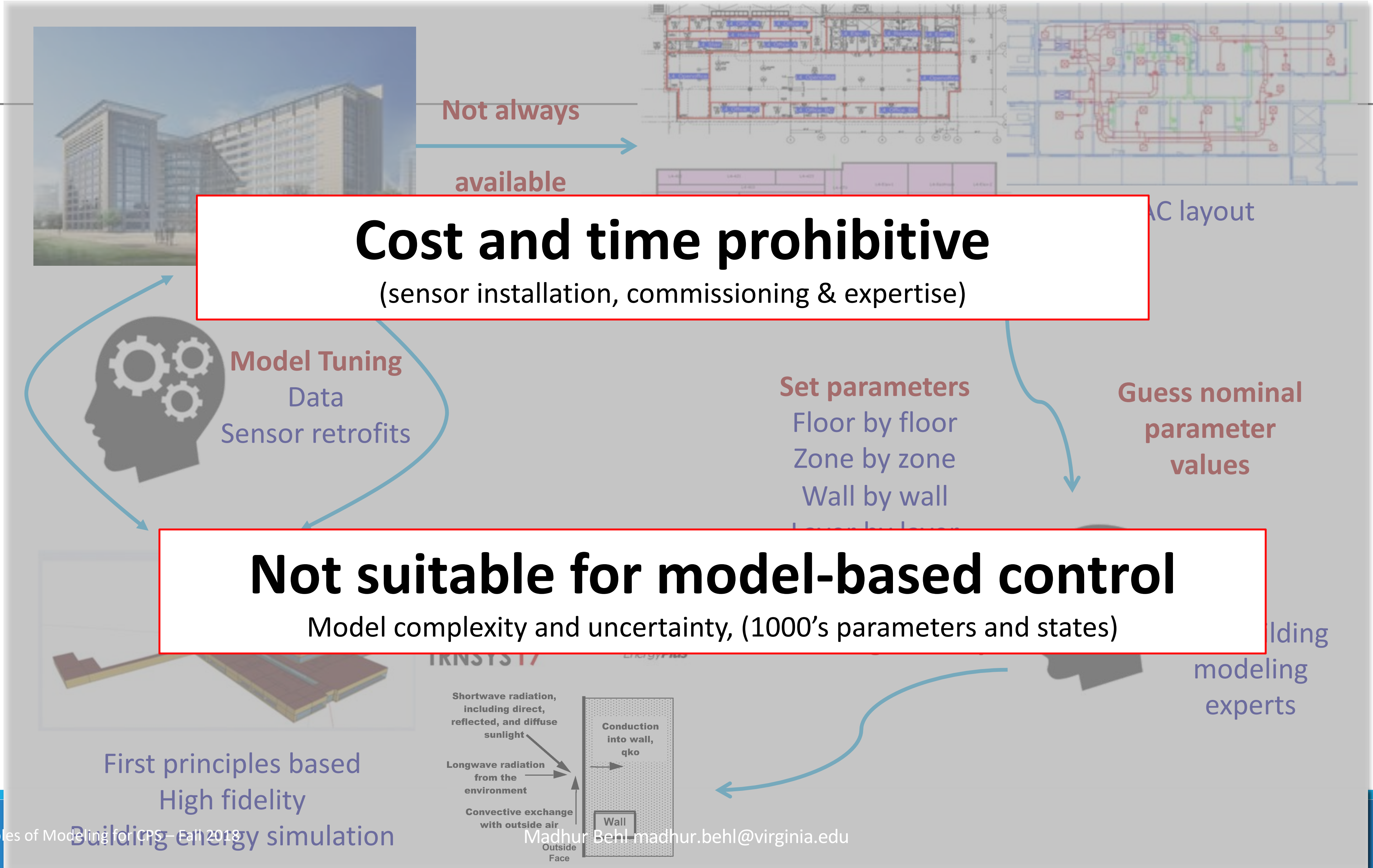
TRNSYS 17

EnergyPlus



First principles based
High fidelity
Building energy simulation

White-Box Modeling



Cost and time prohibitive

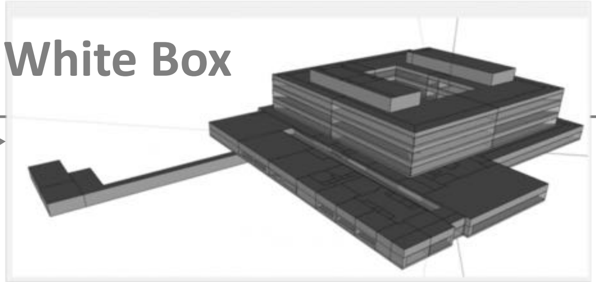
(sensor installation, commissioning & expertise)

Not suitable for model-based control

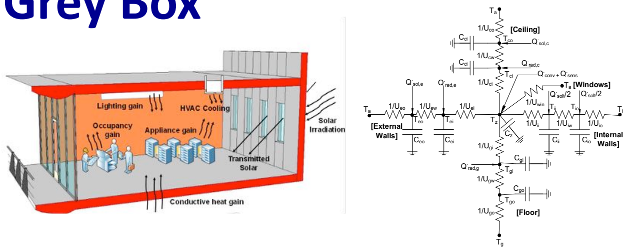
Model complexity and uncertainty, (1000’s parameters and states)

How are building models obtained today ?

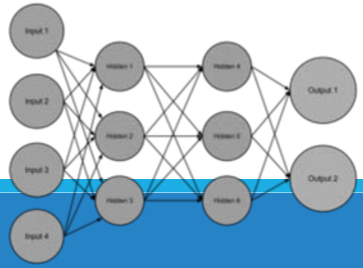
White Box



Grey Box

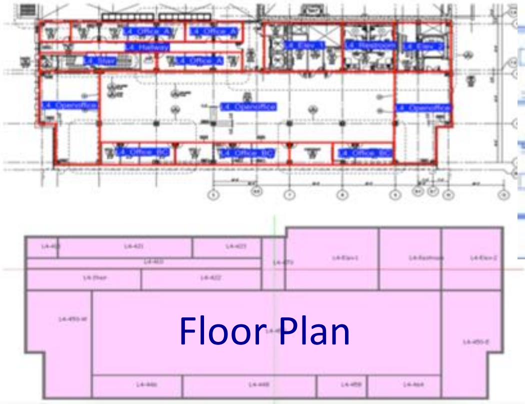


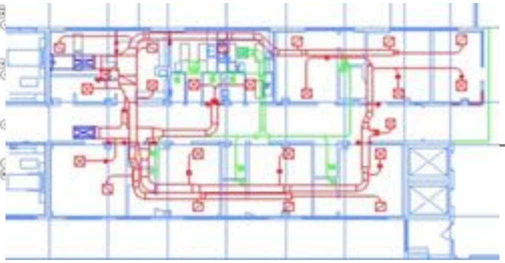
Black Box



Grey-Box [Inverse] Modeling

Not always available





HVAC layout

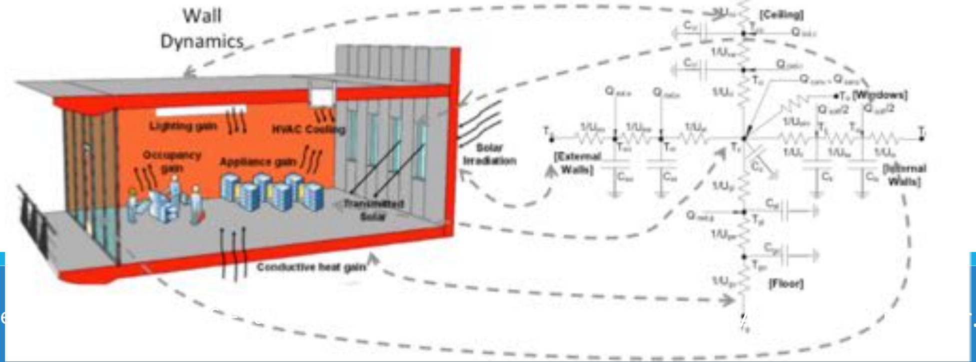
Parameter Estimation
Data
Sensor retrofits

Set parameters
Floor by floor
Zone by zone
~~Wall by wall~~
~~Layer by layer~~
Equipment by equipment
Transfer geometry

Guess nominal parameter values

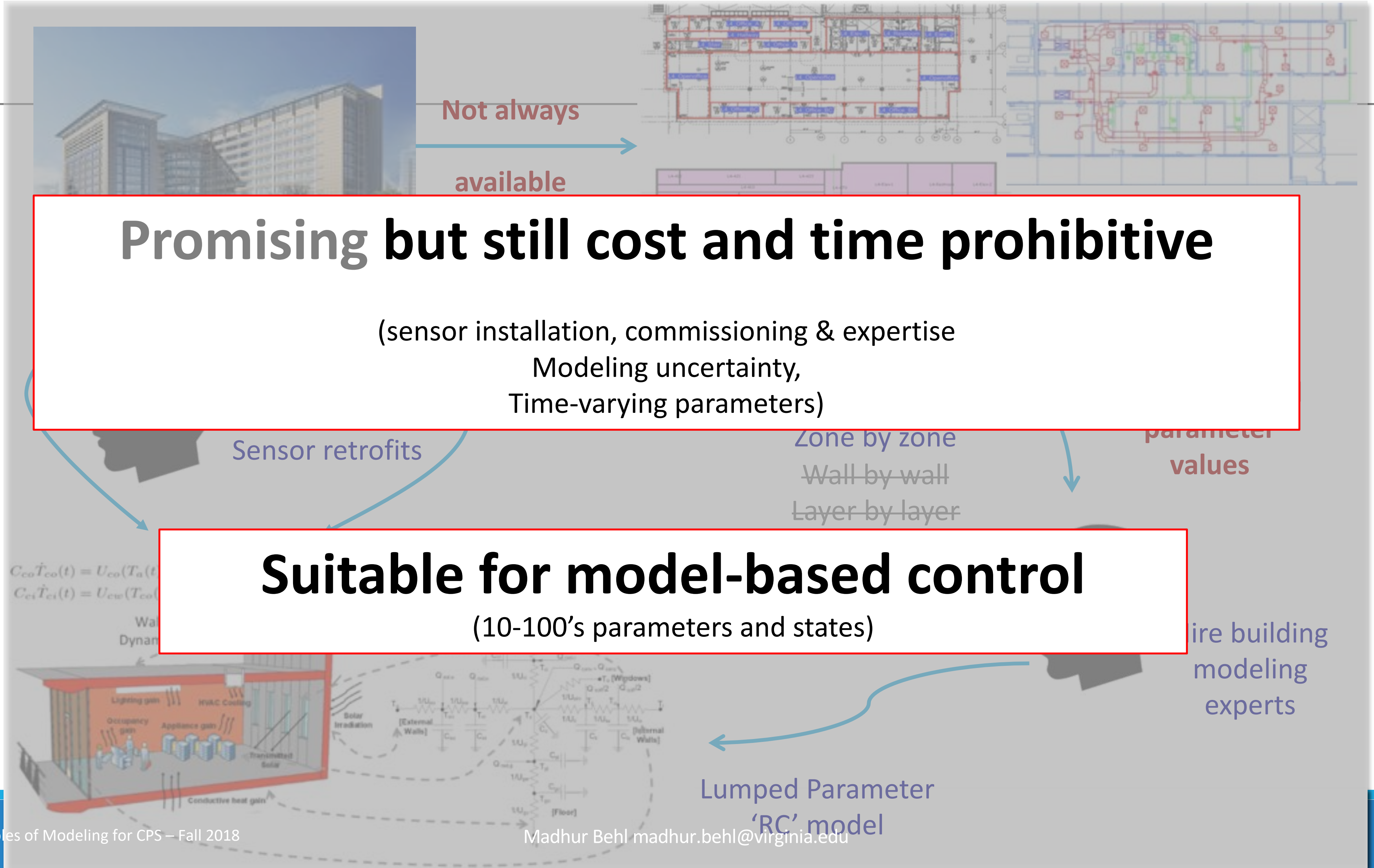
Hire building modeling experts

$$C_{co}\dot{T}_{co}(t) = U_{co}(T_a(t) - T_{co}(t)) + U_{cw}(T_{ci}(t) - T_{co}(t)) + \dot{Q}_{sol,c}(t)$$
$$C_{ci}\dot{T}_{ci}(t) = U_{cw}(T_{co}(t) - T_{ci}(t)) + U_{ci}(T_z(t) - T_{ci}(t)) + \dot{Q}_{rad,c}(t)$$



Lumped Parameter 'RC' model

Grey-Box Modeling



Promising but still cost and time prohibitive

(sensor installation, commissioning & expertise
Modeling uncertainty,
Time-varying parameters)

Suitable for model-based control

(10-100's parameters and states)

Cost and Time prohibitive modeling

OptiControl
Use of weather and occupancy forecasts for optimal building climate control

ETH
Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

SIEMENS

Project duration: May 2007 – March 2015

Phase 1: EnergyPlus model (white-box), RC model (grey box), MPC development and evaluation. [Only simulated studies]

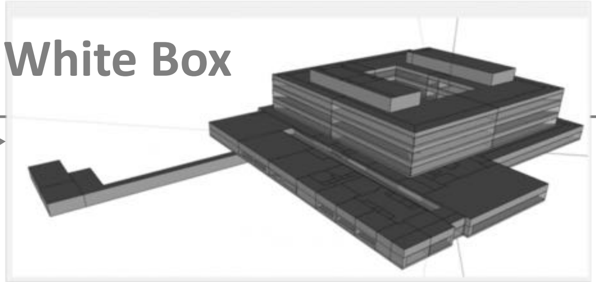
Phase 2: Retrofitted building with sensors, commercial MPC software, demand response, peak reduction, uncertain models..

"..the biggest hurdle to mass adoption of intelligent building control is the cost and effort required to capture accurate dynamical models of the buildings."

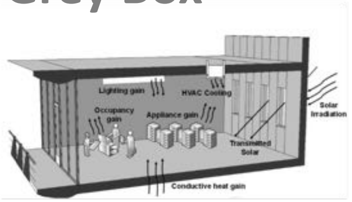
Sturzenegger, D.; Gyalistras, D.; Morari, M.; Smith, R.S., "Model Predictive Climate Control of a Swiss Office Building: Implementation, Results, and Cost-Benefit Analysis," Control Systems Technology, IEEE Transactions on , vol.PP, no.99, pp.1,1, March 2015

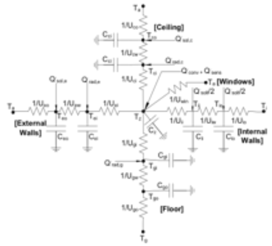
How are building models obtained today ?

White Box

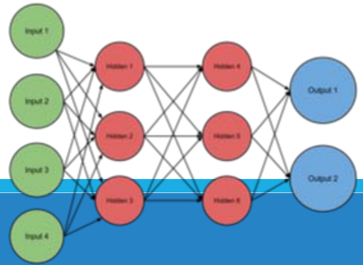


Grey Box





Black Box

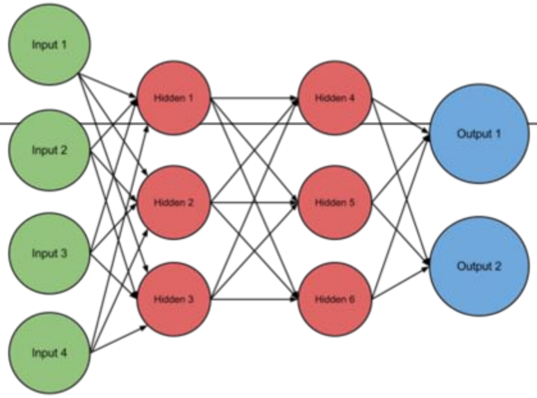


Black-Box Modeling

Data

Feature engineering

Could be automated



Not well aligned with control synthesis

Coarse grained predictions

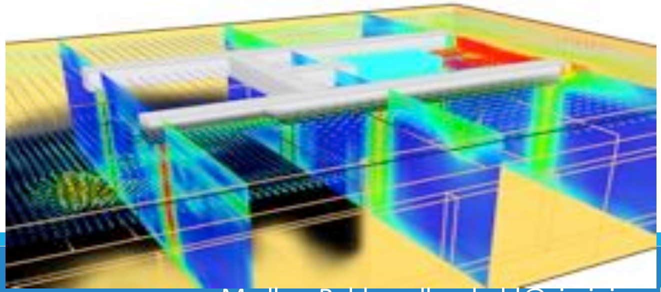
Non-physical parameters

Modeling using first principles is hard !



Each building design is different.
Must be uniquely modeled

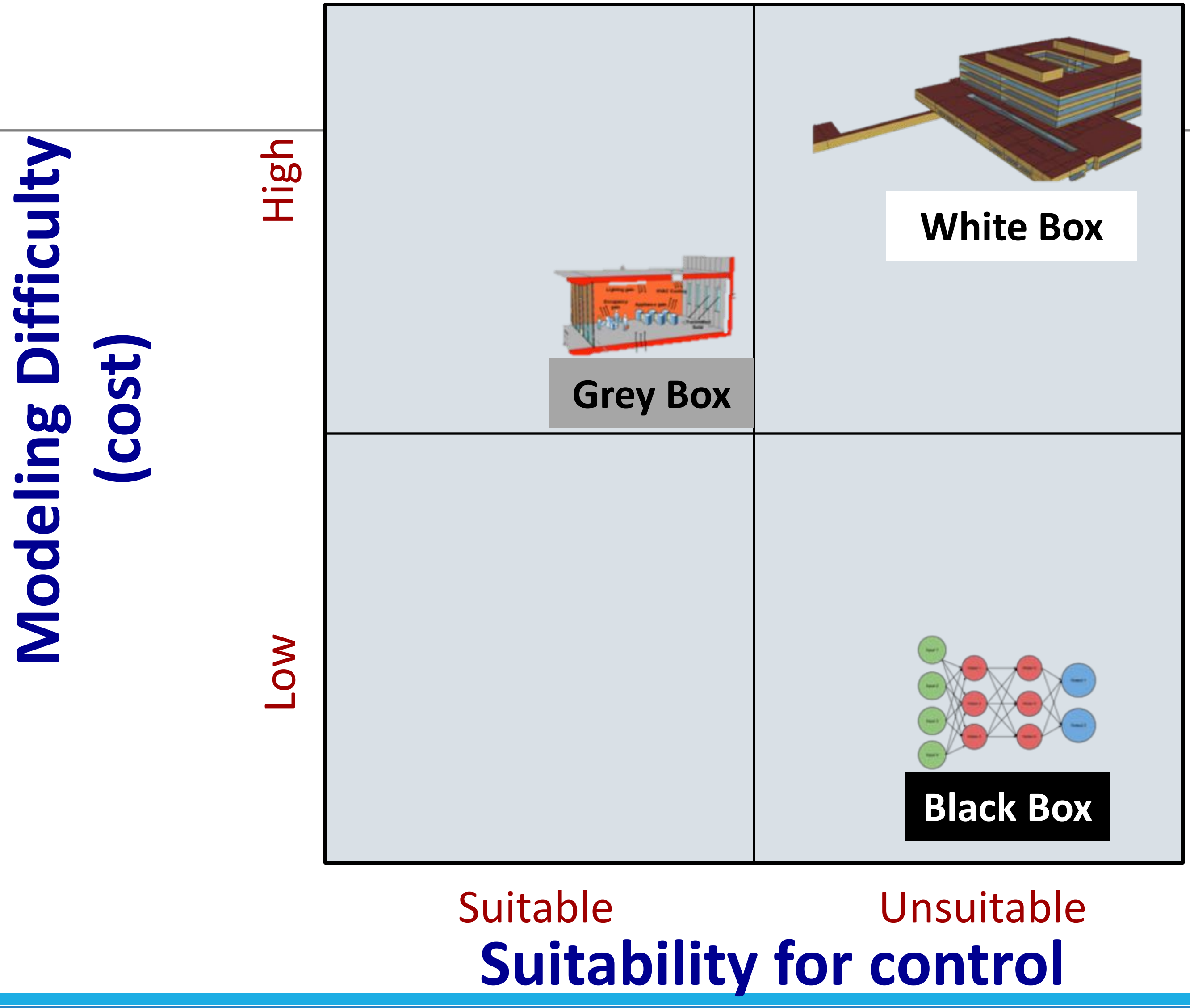

Long operational lifetimes
~50-100 years

Too many sub-systems
Non-linear interactions

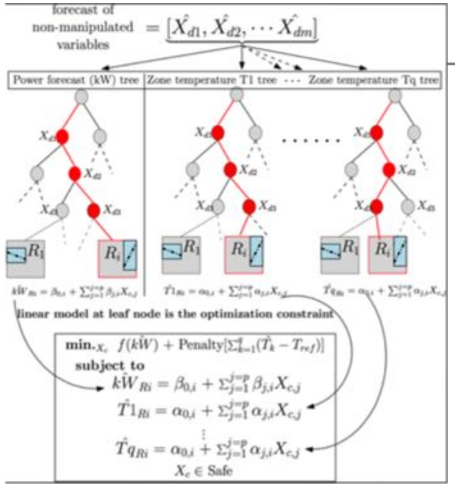
Madhur Behl madhur.behl@virginia.edu

Energy Systems Modeling



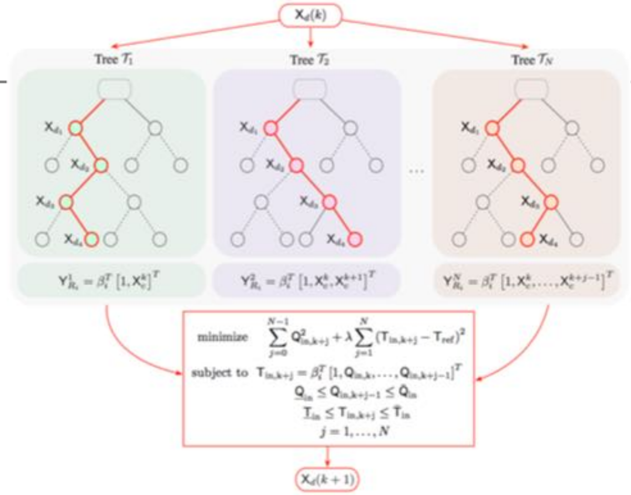
Foundations of Data Predictive Control for CPS

Single-step look ahead
[with single reg. trees]



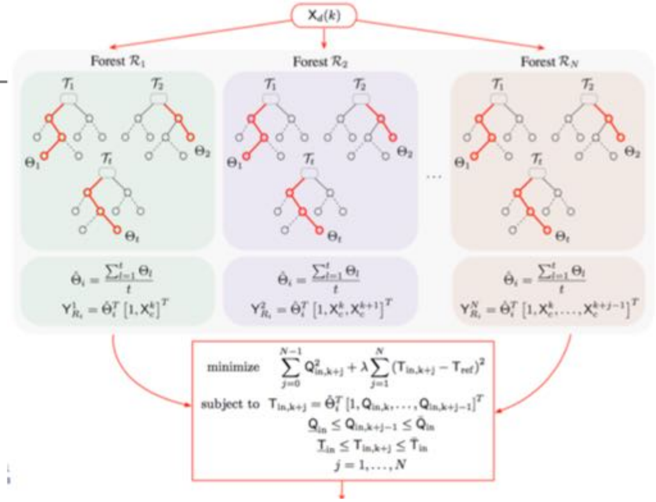
mbCRT

Finite receding horizon
[with single reg. trees]



DPC-RT

Finite receding horizon
[with ensemble models]



Ensemble-DPC

DPC

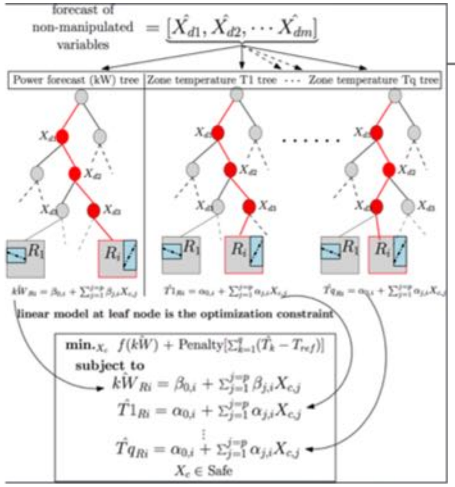
$$\begin{aligned}\text{minimize} \quad & \sum_{j=0}^{N-1} Q_{in,k+j}^2 + \lambda \sum_{j=1}^{N} (T_{in,k+j} - T_{ref})^2 \\ \text{subject to} \quad & T_{in,k+j} = \beta_i^T [1, Q_{in,k}, \dots, Q_{in,k+j-1}]^T \\ & \underline{Q}_{in} \leq Q_{in,k+j-1} \leq \bar{Q}_{in} \\ & \underline{T}_{in} \leq T_{in,k+j} \leq \bar{T}_{in} \\ & j = 1, \dots, N.\end{aligned}$$

MPC

$$\begin{aligned}\text{minimize} \quad & \sum_{j=0}^{N-1} Q_{in,k+j}^2 + \lambda \sum_{j=1}^{N} (T_{in,k+j} - T_{ref})^2 \\ \text{subject to} \quad & x_{k+j} = Ax_{k+j-1} + Bu_{k+j-1} + B_d d_{k+j-1} \\ & \underline{Q}_{in} \leq Q_{in,k+j-1} \leq \bar{Q}_{in} \\ & \underline{T}_{in} \leq T_{in,k+j} \leq \bar{T}_{in} \\ & j = 1, \dots, N\end{aligned}$$

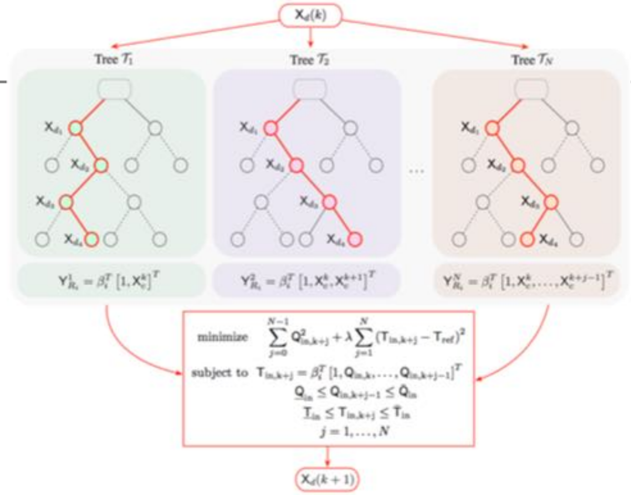
Foundations of Data Predictive Control for CPS

Single-step look ahead
[with single reg. trees]



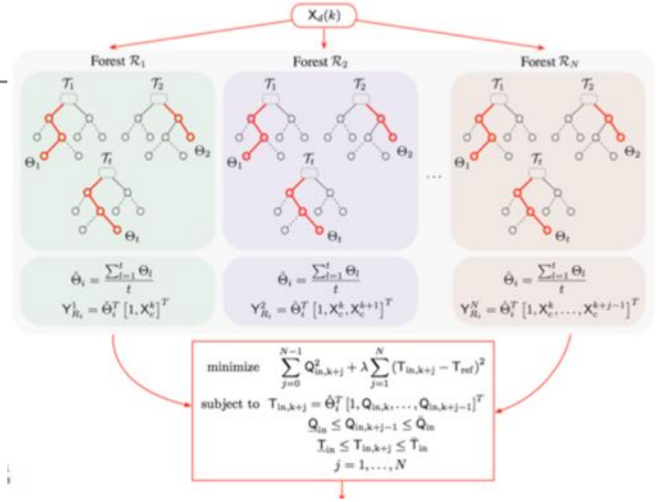
mbCRT

Finite receding horizon
[with single reg. trees]



DPC-RT

Finite receding horizon
[with ensemble models]



Ensemble-DPC

- ICCPS '16, BuildSys 15, CISBAT 15, Journal of Applied Energy
- Best Paper Award (SRC TECHCON-IoT): 'Sometimes, Money Does Grow on Trees'
-Ph.D. Dissertation: Madhur Behl, UPenn (2016)

- ACM BuildSys 16 (Best Presentation Award)
- ACM Transactions of Cyber Physical Systems.

- American Control Conference 17 (*Best Energy Systems Paper Award)*

Energy CPS Module Recap

- ☑ Review of ODEs and dynamical systems.
- ☑ State-Space modeling and implementation in MATLAB, LTI models.
- ☑ First principles – Generalized systems theory.
- ☑ Heat transfer basics.
- ☑ HVAC systems and electricity markets overview.
- ☑ Introduction to EnergyPlus.
- ☑ 'RC' network based state-space thermal modeling.
- ☑ Nominal values of parameters from IDF file.
- ☑ Parameter estimation optimization
- ☑ Non-linear least squares.
- ☑ Model evaluation and goodness of fit.
- ☑ Model sensitivity analysis and experiment design
- ☑ Model predictive control basics
- ☑ Codebase to learn a state-space model from any data-set.