PRINCIPLES OF MODELING FOR CYBER PHYSICAL SYSTEMS

Assignment #6

# Transition Systems and Linear Temporal Logic

Due Date: 11-07-2019 at 2:00pm (Before the lecture)

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The problems have been adapted from the book - Principles of model checking by Christel Baier and Joost-Pieter Katoen.
Remember the following notation:

| | |
|---|---|
| $\Diamond$ | *eventually* |
| $\Box$ | *always* |
| $\bigcirc$ | *next* |
| $\cup$ | *until* |
| $\neg$ | *negation* |
| $\vee$ | *or* |
| $\wedge$ | *and* |

## 1 PROBLEM 1

*(15 points - 2+2+2+3+3+3)*
Consider the following transition system over the set of atomic propositions $\{a, b\}$:
Indicate for each of the following LTL formulae the set of states for which these formulae are fulfilled:

(a) $\bigcirc a$

(b) $\bigcirc\bigcirc\bigcirc a$

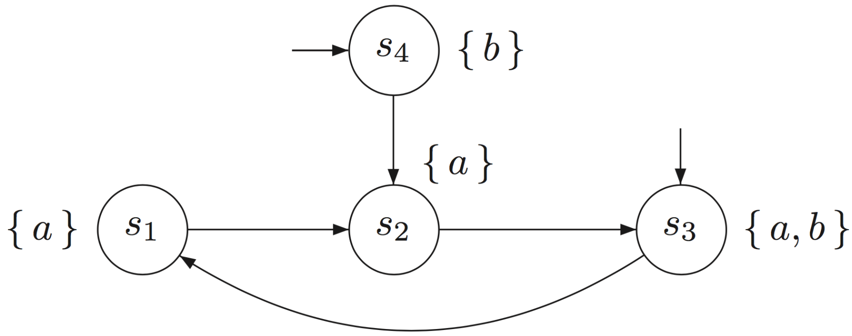


Figure 1.1:

(c) $\Box\, b$

(d) $\Box \Diamond\, a$

(e) $\Box(b \cup a)$

(f) $\Diamond(a \cup b)$

## 2 Problem 2

*(15 points - 2+2+3+2+3+3)*

Consider the transition system $TS$ over the set of atomic propositions $AP = \{a, b, c\}$:

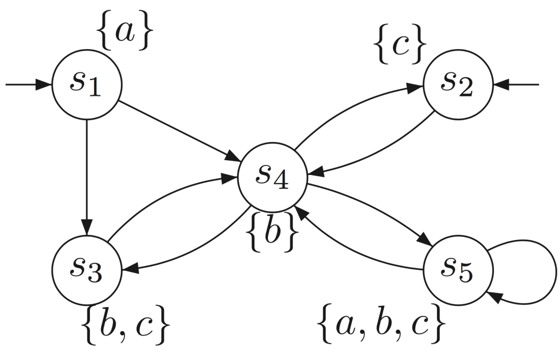


Figure 2.1:

Decide for each of the LTL formulae $\varphi_i$ below, whether $TS \models \varphi_i$ holds. Justify your answers! If $TS \not\models \varphi_i$, provide a path $\pi \in \text{Paths(TS)}$ such that $\pi \not\models \varphi_i$

$$\varphi_1 = \Diamond \Box c$$
$$\varphi_2 = \Box \Diamond c$$
$$\varphi_3 = \bigcirc \neg c \rightarrow \bigcirc \bigcirc c$$
$$\varphi_4 = \Box a$$
$$\varphi_5 = a \cup \Box (b \vee c)$$
$$\varphi_6 = (\bigcirc \bigcirc b) \cup (b \vee c)$$

# 3 PROBLEM 3

*(20 points)*
Suppose we have two users, *Peter* and *Betsy*, and a single printer device *Printer*. Both users perform several tasks, and every now and then they want to print their results on the *Printer*. Since there is only a single printer, only one user can print a job at a time. Suppose we have the following atomic propositions for *Peter* at our disposal:

- *Peter.request* ::= indicates that *Peter* requests usage of the printer;
- *Peter.use* ::= indicates that *Peter* uses the printer;
- *Peter.release* ::= indicates that *Peter* releases the printer.

For *Betsy*, similar predicates (propositions) are defined. Specify in LTL the following properties:

(a) Mutual exclusion, i.e., only one user at a time can use the printer.

(b) Finite time of usage, i.e., a user can print only for a finite amount of time.

(c) Absence of individual starvation, i.e., if a user wants to print something, he/she eventually is able to do so.

(d) Absence of blocking, i.e., a user can always request to use the printer

(e) Alternating access, i.e., users must strictly alternate in printing.

# 4 PROBLEM 4

*(20 points)*
Consider an elevator system that services $N > 0$ floors numbered 0 through $N-1$. There is an elevator door at each floor with a call-button and an indicator light that signals whether or not the elevator has been called. For simplicity consider $N = 4$. Present a set of atomic propositions - try to minimize the number of propositions - that are needed to describe the following properties of the elevator system as LTL formulae and give the corresponding LTL formulae:

(a) The doors are "safe", i.e., a floor door is never open if the elevator is not present at the given floor.

(b) A requested floor will be served sometime.

(c) Again and again the elevator returns to floor 0.

(d) When the top floor is requested, the elevator serves it immediately and does not stop on the way there.

# 5 PROBLEM 5

*(30 points - 10 + 10 + 5 + 5)*
Which of the following equivalences *(if and only if)* are correct? Prove the equivalence or provide a counterexample that illustrates that the formula on the left and the formula on the right are not equivalent.

(a) $\Box\varphi \rightarrow \Diamond\psi \;\equiv\; \varphi \cup (\psi \vee \neg\varphi)$

(b) $\Diamond\Box\varphi \rightarrow \Box\Diamond\psi \;\equiv\; \Box(\varphi \cup (\psi \vee \neg\varphi))$

(c) $\Box\Box(\varphi \cup \neg\psi) \;\equiv\; \neg\Diamond(\neg\varphi \wedge \psi)$

(d) $\Diamond(\varphi \wedge \psi) \;\equiv\; \Diamond\varphi \wedge \Diamond\psi$