PRINCIPLES OF MODELING FOR CYBER PHYSICAL SYSTEMS

Assignment #2

Thermal 'RC' Network based State-Space Building Modeling

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1 CREATING THE MODEL STRUCTURE

You will create a state-space dynamical model of a single zone in a building. This involves,

Problem 1: creating the lumped parameter thermal RC network for the zone,

Problem 2: writing the heat balance equations for different nodes and branches in the RC network,

Problem 3: identifying the states, disturbances, control inputs, and output variables of interest, and

Problem 4: constructing a continuous time state-space model representation of the RC network.

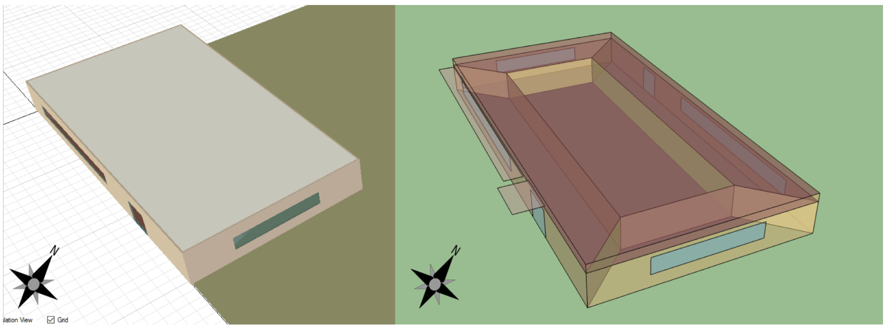

Figure 1.1: Single story building with 5 zones, located in Chicago, IL

BUILDING DESCRIPTION

LAYOUT The building under consideration is a 1 story building. (Fig 1.1) The building is located in Chicago, IL. The building is oriented 30 degrees east of north. The total floor area is 463.6m^2 (100 × 50)ft. It is divided into 4 exterior/peripheral and one interior conditioned zones and a return plenum. The zone height is 8 feet and the return plenum height is 2 feet. The overall building height is 10 feet. There are windows on all 4 facades; the south and north facades have glass doors as well. The window to wall ratio is approximately 0.29.

INTERNAL HEAT GAINS

- The heat gain due to lighting (when fully ON) is $1.5W/ft^2$.
- Office equipment adds to the internal heat gain at $1W/ft^2$.
- There is one occupant per $100ft^2$ of floor area.

HVAC The HVAC system for the building comprises of a standard variable air volume (VAV) system with outside air, hot water reheat coils, central chilled water cooling coil. The central HVAC plant is single hot water boiler, electric compression chiller with air cooled condenser. All equipment is autosized. Hot water (HW) and chilled water (ChW) coils are used in the outside air stream to precondition the outside air.

2 PROBLEM 2-1: SINGLE ZONE RC NETWORK.

Create the thermal RC network structure for the north facing exterior/perimeter zone (space `SPACE3-1`) of this building, as depicted below in the plan view of the building (Fig 2.1).
For the `SPACE3-1` zone, assume that the following sensor measurements are available (use the notation provided for these measurements in your model structure.):

1. Ground Temperature, T_g (°C)

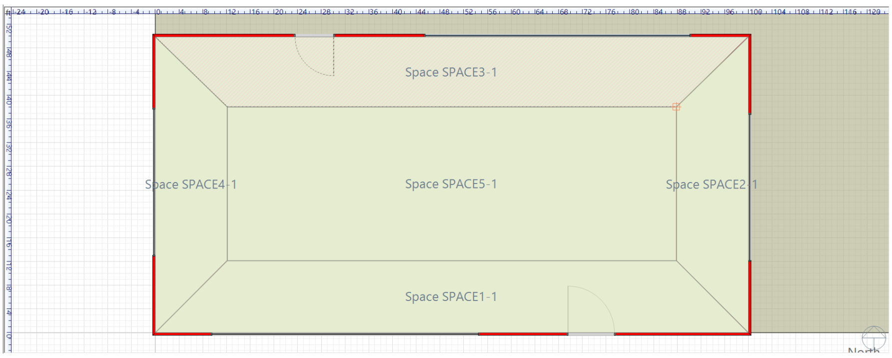

Figure 2.1: `SPACE3-1` is the north facing exterior zone of the building.

2. Outside ambient temperature, T_a (°C)
3. Return air plenum temperature, T_p (°C)
4. Total external solar heat gain, $Q_{sol,e}$ (W)
5. Total internal heat gain, Q_{gain} (W)
6. Total sensible cooling load, Q_{cool} (W)
7. Neighboring zone temperatures for `SPACE2-1` T_2, `SPACE4-1` T_4, `SPACE5-1` T_5 (°C)
8. SPACE3-1 zone temperature, T_z (°C)

Carefully observe the external walls, internal surfaces (neighboring spaces/zones), and glass doors/windows of zone `SPACE3-1` since these are the primary surfaces in this zone. The floor and the ceilings cannot be viewed in the floor plan layout but should be accounted for in the RC network.(Fig 2.1)

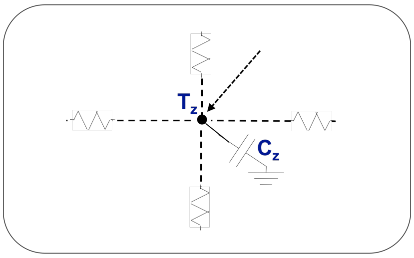

Figure 2.2: Use this simple template to get started and complete the RC graph for zone `SPACE3-1`.

Submit the following: (25 points)

- **[P2-1.a]** Draw and submit the RC network model structure for the zone under consideration. Start with the simple RC network structure (Fig 2.2) and add elements (thermal resistance, thermal capacitance, thermal nodes) to build the RC graph for zone `SPACE3-1`.
- **[P2-1.b]** Clearly label all the elements of the RC network. The RC network should make use of the sensor measurements stated above for the zone `SPACE3-1`. You can refer to lecture slides for consistent notation of thermal elements.

3 Problem 2-2: Differential equations for thermal dynamics

Submit the following: (30 points)

- **[P2-2.a](20)** For each branch of the RC network, using principles of heat transfer covered in the lectures, write down the heat balance differential equations which describe the rate of change of temperature for all the states/nodes of the RC network constructed in Problem 1.
- **[P2-2.b](10)** How many ODE equations do you have ? Is there a relation between the RC network structure and the number of ordinary differential equations ODE's ?
- **[P2-2.c](10)** List all the thermal resistance/conductance and capacitance parameters that exist in your dynamical equations.

4 **PROBLEM 2-3:** STATES, INPUTS, AND OUTPUTS

Submit the following: (15 points) Based on the ODEs from Problem 2-2, determine the following:

- **[P2-3.a](5)** How many state variables exist in the system ? Does this answer differ from the answer to Problem P2-2.b (Y/N)?
- **[P2-3.b](4)** Which input variables constitute as disturbances for your model (list all) ?
- **[P2-3.c](3)** What are the control input(s) for your model (list all) ?
- **[P2-3.d](3)** What are the output variable(s) in your model (list all) ?

5 **PROBLEM 2-4:** STATE SPACE MODEL

Rewrite the differential equations for your model in a continuous-time state-space representation.

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) \\ y(t) &= Cx(t) + Du(t) \end{aligned} \tag{5.1}$$

,
where the state $x(t)$ is a vector of all node temperatures of the model, and the input $u(t)$ is a vector of all the inputs to the system. Follow convention and ensure that the last element in the input vector is the control input.

Submit the following: (30 points)

- **[P2-4.a](10)** Write the state vector$x(t)$ and its elements, the input vector $u(t)$ and its elements, and the output vector $y(t)$ and its elements for your model.
- **[P2-4.b](10)** Show all the elements of the A, B, C, and D matrices. How do these matrices depend on the RC parameters (linearly or non-linearly) ?
- **[P2-4.c](10)** Summarize your state-space model and provide the following:

1. What is the model order for your state-space model ?
2. How many inputs variables are present in your state-space model ?
3. How many outputs are present in your state-space model ?
4. How many total parameters are present in your state-space model ?
5. What are the dimensions of the A,B,C, and D matrices for your state-space model ?